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ESD-TDR-63-198  
Vol II

# PERT EACH

(PERT Fundamentals, Volume II)

TECHNICAL DOCUMENTARY REPORT NO. ESD-TDR-63-198

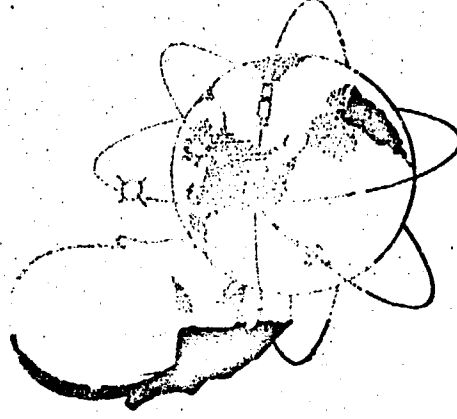
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OPERATIONAL APPLICATIONS LABORATORY  
DEPUTY FOR TECHNOLOGY  
ELECTRONIC SYSTEMS DIVISION  
AIR FORCE SYSTEMS COMMAND  
L. G. Hanscom Fld., Bedford, Mass

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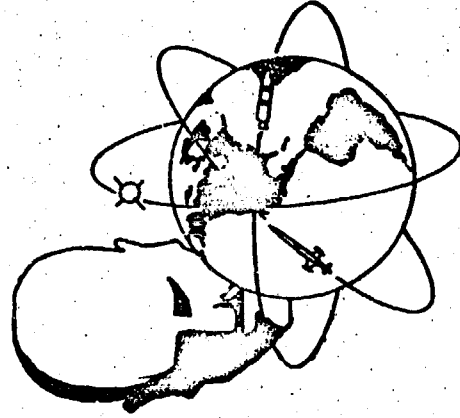
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NOTE: Instructions for this volume  
and all other volumes are to  
be found in Volume I.

ESD-TDR-63-198

PERTeach

ABSTRACT

This self-instructional course teaches the basic concepts and techniques of PERT (Program Evaluation Review Technique). The course consists of six volumes and is intended for use by Air Force managers. Presented in programmed-instruction format, the course allows the student to proceed at his own pace and to learn without the aid of an instructor.

PUBLICATION REVIEW AND APPROVAL

This Technical Documentary Report has been reviewed and is approved.

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## VOLUME II

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PERTeach

Volume II

CHAPTER I

Latest Time

Suppose you can expect that from your office it will take you exactly one-half hour to reach a train that starts at 5:00 p.m. To catch this train you can leave your office at 3:30 p.m., at a later time, 4:00 p.m., or at an even later time, 4:15 p.m. But unless you delay the train, 4:30 p.m. is the \_\_\_\_\_ at which you can leave your office.

Latest Time

Each event of a PERT network has a characteristic called its Latest Time,

TL.

There is a \_\_\_\_\_, TL, for each \_\_\_\_\_ of a PERT network.



Latest Time

event

The symbol for Latest Time is \_\_\_\_\_.

$T_L$

---

The  $T_L$  of an event is the latest time at which it can take place without delaying occurrence of the network ending event beyond a specified time.

$T_L$  is the symbol for \_\_\_\_\_.

### Latest Time

The Accumulated Expected Time,  $T_E$ , of any event is the time at which the event can be expected to occur after the network beginning event has taken place.  $T_E$  is related only to the network beginning event.

The Latest Time,  $T_L$ , of any event is the latest time at which it can take place without delaying occurrence of the network ending event.  $T_L$  is related only to the network ending event.

The  $T_L$  value of any event \_\_\_\_\_ (need not/must always) equal its  $T_E$  value.

need not

Scheduling or contract requirements may specify the time or date when a network is to be completed; that is, when the network ending event is to take place. This specified time may differ from the Accumulated Expected Time,  $TE$ , of the network ending event. Where no contractual or schedule dates for completion of the network exist, the Latest Times of its events are calculated with respect to the  $TE$  of the network ending event.

For the remainder of this chapter we will assume that there are no contractual or scheduling requirements that specify when the network must be completed.

Assuming that there are no contractual or scheduling requirements, the  $T_L$  value of an event is calculated with respect to the  $T_E$  value of the network ending event. Then the Latest Time,  $T_L$ , of an event is the latest time it can take place without delaying occurrence of the \_\_\_\_\_ beyond its  $T_E$  value.

network ending event

Assuming that there are no contractual or scheduling agreements, the Latest Time,  $T_L$ , of all events are calculated with respect to the  $T_E$  of the network ending event. Therefore, before  $T_L$  values can be calculated, the  $T_E$  of the network ending event must be known.

Suppose the  $T_E$  of a network ending event is 61.0 weeks. Then, the  $T_L$  of any event is the latest time it can take place without delaying occurrence of the \_\_\_\_\_ beyond \_\_\_\_\_ weeks.

network ending event beyond 61.0 weeks.

\_\_\_\_\_

The  $T_L$  of an event is the latest time it can take place without delaying the network ending event beyond its  $T_E$  value. Any delay in occurrence of the network ending event itself will make it take place later than its  $T_E$  value.

Therefore the \_\_\_\_\_ value of the network ending event must equal its \_\_\_\_\_ value.

$T_E$   $T_L$   
or  
 $T_L$   $T_E$

---

The  $T_L$  of a network ending event equals the previously determined  $T_E$  value of this event.

If the  $T_E$  value of a network ending event is 61.0 weeks, its \_\_\_\_\_ value equals \_\_\_\_\_ weeks.



TL  
61.0

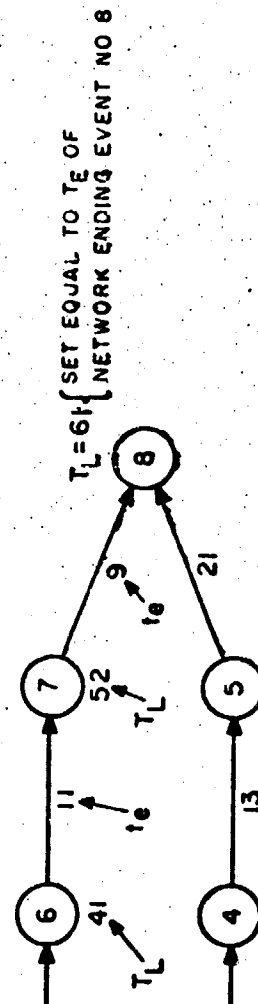
2-12

For any activity, the  $T_L$  of its beginning event equals the  $T_L$  of its ending event minus the  $t_e$  value of the activity. Start  $T_L$  calculations at the network ending event.

In the network below, event no. 8 is the ending event of activity 7-8 as well as the network ending event. Event no. 7 is the beginning event of activity 7-8 as well as the ending event of activity 6-7. Assume  $T_E$  of event no. 8 equals 61 weeks.

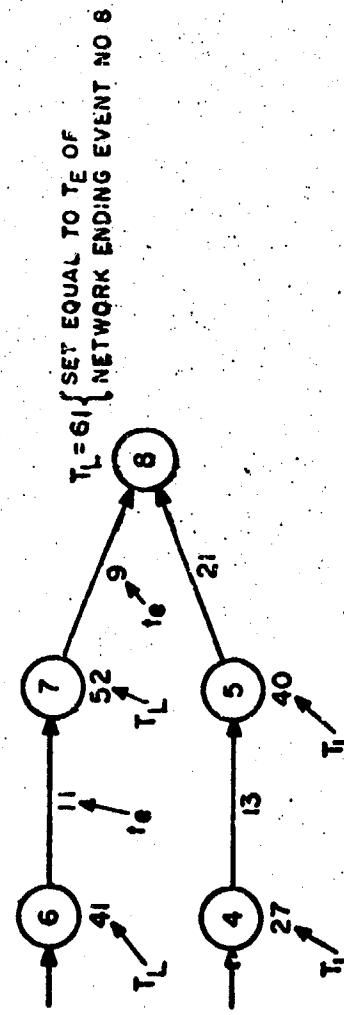
Following the above rule,  $T_L$  of event no. 7 equals (61-9) or 52 and the  $T_L$  of event no. 6 equals (52-11) or 41.

The  $T_L$  of event no. 5 is \_\_\_\_\_. The  $T_L$  of event no. 4 is \_\_\_\_\_.



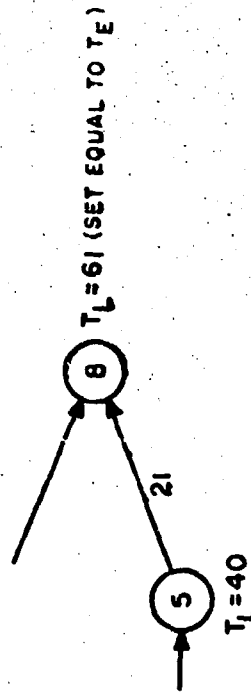
$T_L$  of event no. 5 is 40.

$T_L$  of event no. 4 is 27.



In the network below, the  $T_L$  of event no. 5 is 40. Adding the  $t_e$  of activity 5-8 (21) to 40 gives 61, the  $T_L$  of event no. 8. If event no. 5 occurs at any time later than 40, adding 21 to it will make event no. 8 occur at a time \_\_\_\_\_ (earlier/ later) than its  $T_L$  value.

Thus 40 is the \_\_\_\_\_ at which event no. 5 can take place without delaying the occurrence of the network ending event, no. 8, beyond 61.



later  
latest time

2-16

Always start  $T_L$  calculations at the network \_\_\_\_\_ event.

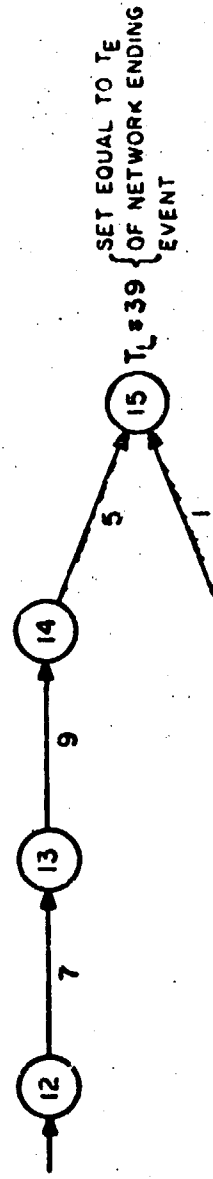
For any activity, the  $T_L$  of its beginning event equals the  $T_L$  of its ending event minus the  $t_e$  of the activity.

In the network shown below:

$T_L$  of event no. 14 = \_\_\_\_\_

$T_L$  of event no. 13 = \_\_\_\_\_

$T_L$  of event no. 12 = \_\_\_\_\_

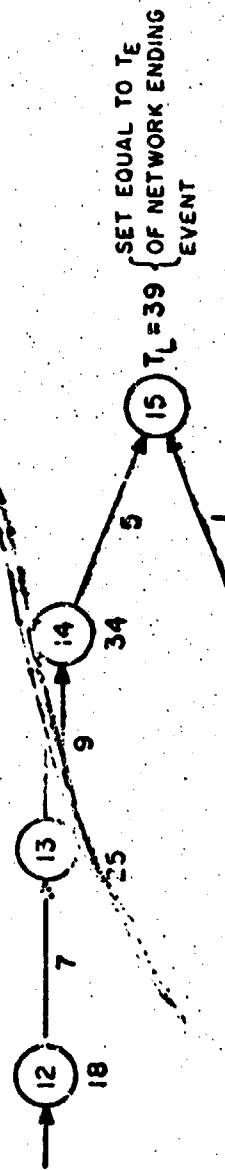


ending

$T_L$  of event no. 14 = 34

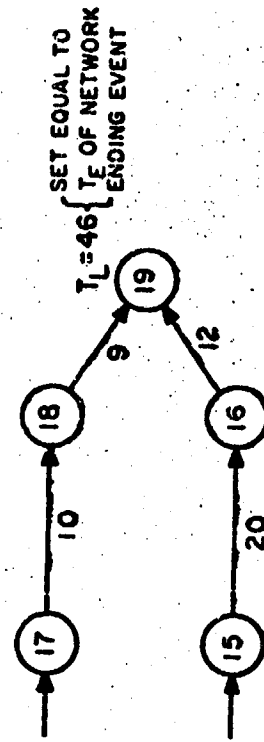
$T_L$  of event no. 13 = 25

$T_L$  of event no. 12 = 18

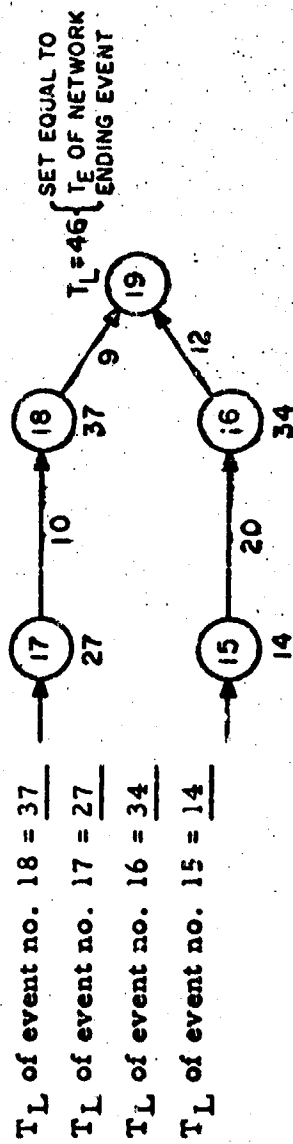


In the network shown:

- $T_L$  of event no. 18 = \_\_\_\_\_  
 $T_L$  of event no. 17 = \_\_\_\_\_  
 $T_L$  of event no. 16 = \_\_\_\_\_  
 $T_L$  of event no. 15 = \_\_\_\_\_



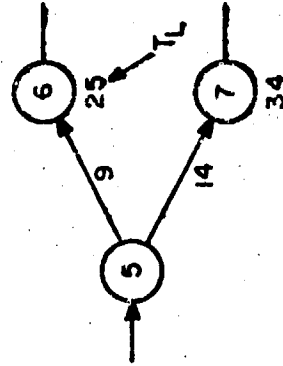




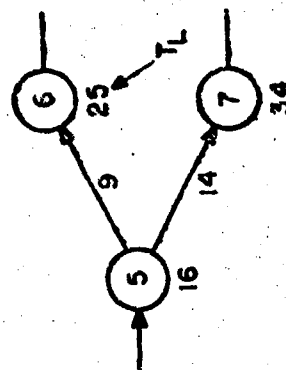
Where an event is the beginning event for more than one activity, its  $T_L$  value is the smallest of all possible values.

In the section of network below, event no. 5 is the beginning event of activities 5-6 and 5-7. One possible value of the  $T_L$  for event no. 5 is 16. This is obtained by subtracting 9, the  $t_e$  of activity 5-6 from 25, the  $T_L$  of event no. 6. The other possible value, obtained in similar manner with activity 5-7, is (34-14) or 20.

Since 16 is smaller than 20, the  $T_L$  of event no. 5 is \_\_\_\_.

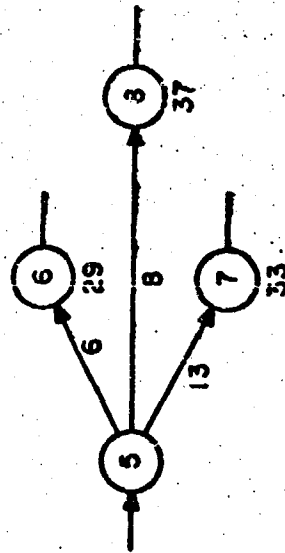


The  $T_L$  of event no. 5 is 16.

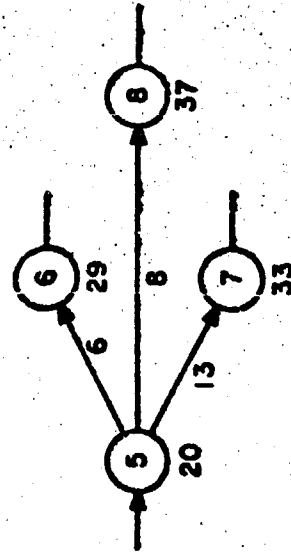


In the section of network below, event no. 5 is the beginning event of activities 5-6, 5-7 and 5-8.

The  $T_L$  of event no. 5 is \_\_\_\_.



The  $T_L$  of event no. 5 is 20.

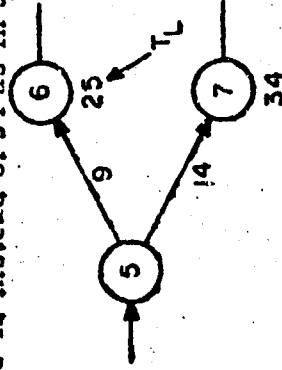


The  $T_L$  of a beginning event that starts more than one activity must be the smallest of all possible values. Otherwise, some events which are to occur after the given event will take place at times later than their calculated  $T_L$  values.

In the network below, the possible  $T_L$  values of event no. 5 are 16 and 20. If 20 is used as the  $T_L$  value, and we add to it the  $t_e$  (14) of activity 5-7, then we allow event no. 7 to occur at a time no later than its  $T_L$  value, 34. However, adding the  $t_e$  (9) of activity 5-6 to 20 will force event no. 6 to occur at 29, a time later than its  $T_L$  value, 25.

If 16 is used as the  $T_L$  value of event no. 5, adding 16 to 9 ( $t_e$  of activity 5-6) permits event no. 6 to occur no later than its  $T_L$  value. Also adding 16 to 14 ( $t_e$  of activity 5-7) allows event no. 7 to take place before its  $T_L$  value.

Suppose the  $T_L$  of event no. 7 were 42 instead of 34 as in the sketch - would that change the  $T_L$  of event no. 5?



Your Answer \_\_\_\_\_  
 A. Yes  
 B. No.

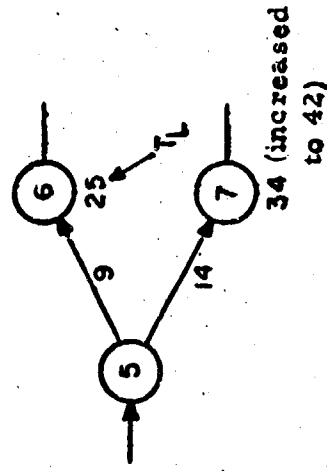
Turn to page  
 2-26  
 2-27

Your Answer: A. Yes

Wrong. If the increased  $T_L$  of event no. 7 is 42, subtracting from it the  $t_e$  of activity 5-7 gives (42-14) or 28. The  $T_L$  of event no. 6 remains unchanged at 25. Subtracting the  $t_e$  of activity 5-6 from this leave 16,

Remember that the  $T_L$  of an event must be the smallest of possible values.

Return to page 2-25, review the information, then select the correct answer.

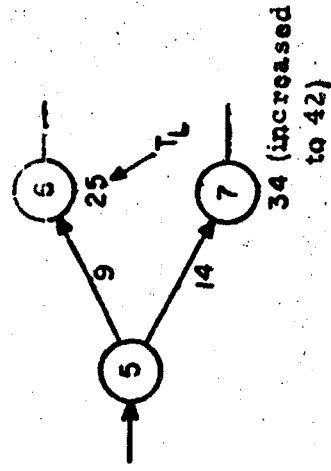


**Your Answer:** A. Yes

Wrong. If the increased  $T_L$  of event no. 7 is 42, subtracting from it the  $t_e$  of activity 5-7 gives (42-14) or 28. The  $T_L$  of event no. 6 remains unchanged at 25. Subtracting the  $t_e$  of activity 5-6 from this leaves 16.

Remember that the  $T_L$  of an event must be the smallest of possible values.

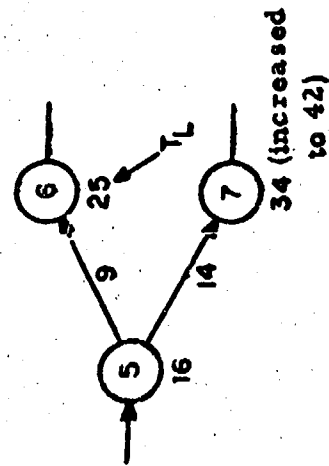
Return to page 2-25, review the information, then select the correct answer.





Your Answer: B. No

Correct! Subtracting the  $t_e$  (14) of activity 5-7 from the increased  $T_L$  (42) of event no. 7 gives 28, a value greater than 16, the value obtained with event no. 6 and activity 5-6. The  $T_L$  of event no. 5 is the smaller of these two values and remains unchanged at 16.



Turn to next page.

For any activity,  $T_L$  of the \_\_\_\_\_ event equals  $T_L$  of the \_\_\_\_\_ event minus  $t_e$  of the activity.

Where an event is the beginning event for more than one activity, its  $T_L$  value is the \_\_\_\_\_ (smallest/largest) of all possible  $T_L$  values.

For any activity,  $T_L$  of the beginning event equals  $T_L$  of the ending event minus  $t_e$  of the activity.

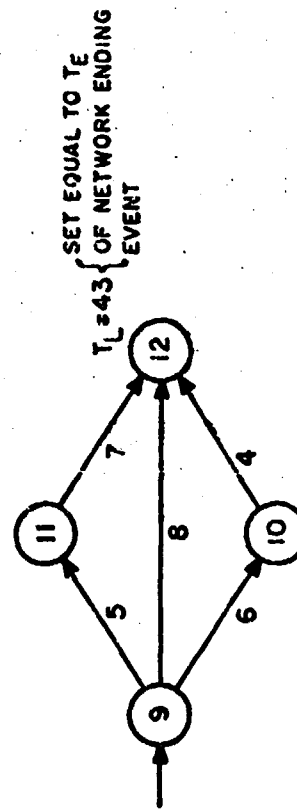
$T_L$  is the smallest of all possible  $T_L$  values.

In the following network section:

$T_L$  of event no. 11 = \_\_\_\_\_

$T_L$  of event no. 10 = \_\_\_\_\_

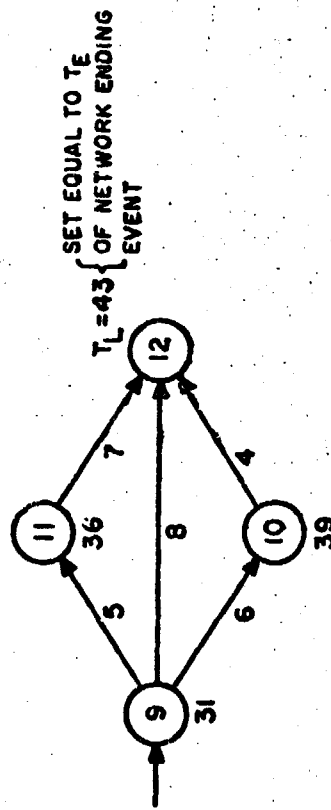
$T_L$  of event no. 9 = \_\_\_\_\_



$T_L$  of event no. 11 = 36

$T_L$  of event no. 10 = 39

$T_L$  of event no. 9 = 31



We must know the  $T_L$  of the network ending event when we start all calculations of  $T_L$  values. The  $T_L$  of the network ending event is set equal to its  $T_E$  value.

Therefore before we start  $T_L$  calculations we must determine the \_\_\_\_ value of the \_\_\_\_.

T<sub>E</sub> value of the network ending event.

---

When you determined the T<sub>E</sub> of the network ending event to be 66.0 in Panel B you also found the T<sub>E</sub> values of all other events of the network. However, when you found T<sub>L</sub> values in the last few pages you did not know the T<sub>E</sub> of any event except the network ending event, and in these examples the network beginning event was not even shown.

Except for the network ending event, the T<sub>E</sub> of a given event \_\_\_\_\_ (must be/is not) used to determine the T<sub>L</sub> of the given event.

is not

To say that a given event has a  $T_L$  of 29.0 weeks means that the event must occur no later than 29.0 weeks after the network beginning event.

$T_L$  is expressed as time elapsed after occurrence of the Network \_\_\_\_\_ Event but calculation of  $T_L$  starts at the Network \_\_\_\_\_ Event.



beginning  
ending

2-35

1 2 3 4 5 6 7 8 9 10 11 12 13 14 15 16 17 18 19 20 21 22 23 24 25 26 27 28 29 30 31 32 33 34 35 36 37 38 39 40 41 42 43 44 45 46 47 48 49 50 51 52 53 54 55 56 57 58 59 60 61 62 63 64 65 66 67 68 69 70 71 72 73 74 75 76 77 78 79 80 81 82 83 84 85 86 87 88 89 90 91 92 93 94 95 96 97 98 99 100

Turn to Panel C in the workbook and leave it exposed to view. For computation of  $T_L$  values in a complete network, a table like that of Panel C is convenient. To simplify later use of  $T_L$  values in this course, the events are arranged with the network beginning event at the top of the table and the network ending event at the bottom. Because  $T_L$  calculations start at the network ending event, we start at the bottom of the table and work up towards the top.

For these computations we assume that all activity  $t_e$  values are known and that the  $T_L$  of the network ending event (equal to its  $T_E$  value) has been determined by previous  $T_E$  calculations.

We will use the relation that for any activity, the  $T_L$  of its beginning event equals the  $T_L$  of its ending event minus the  $t_e$  of the activity. Also, if an event is the beginning event for more than one activity, its true  $T_L$  value is the smallest of all possible values.

$$T_L (\text{of beginning event}) = T_L (\text{of ending event}) - t_e (\text{of activity})$$

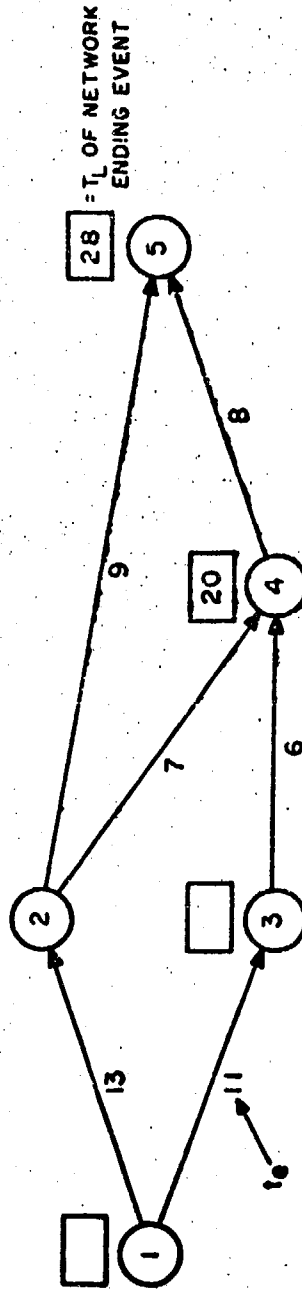
Start filling in the blanks in the table and network of Panel C in accordance with the following directions:

Step 1. In column labelled "Ending Event  $T_L$ " and in the bottom row, next to activity 4-5, write the known  $T_L$  of network ending event no. 5. This  $T_L$  is given in the network. Then subtract the  $t_e$  of activity 4-5 from the  $T_L$  of event no. 5. Write the result, a possible  $T_L$  of event no. 4, in the column labelled "Beginning Event Possible  $T_L$ ".

Look at event no. 4 in the network. It is a beginning event for only one activity, 4-5. Therefore the possible  $T_L$  which you just calculated for event no. 4 must be its true  $T_L$ . Place a star beside this  $T_L$  value in the table.

Finally, write the  $T_L$  of event no. 4 in the box over event no. 4 in the network of Panel C.

Your Panel C should now look like this. If it doesn't correct it.

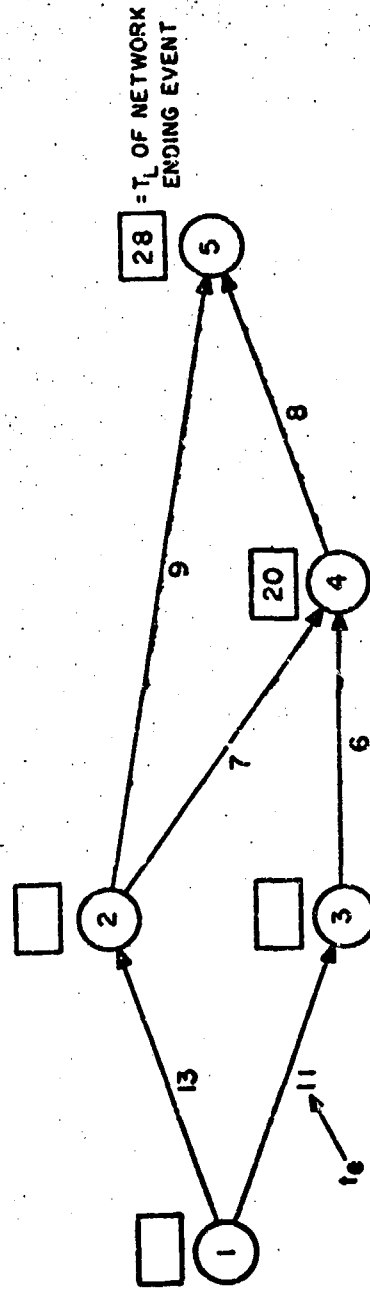


	ACTIVITY		ACTIVITY $t_o$	ENDING EVENT $T_L$	BEGINNING EVENT POSSIBLE $T_L$
	BEGIN EVENT	ENDING EVENT			
STEP 6	1	2	13		
STEP 5	1	3	11		
STEP 4	2	4	7		
STEP 3	3	4	6		
STEP 2	2	5	9		
STEP 1	4	5	8	28	20*
* = TRUE $T_L$ OF BEGINNING EVENT					

Step 2. In Panel C, write the  $T_L$  of event no. 5 again in the Ending Event  $T_L$  column but this time in line with activity 2-5. Subtract the  $t_e$  of activity 2-5 from this  $T_L$  value. Write the result, a possible  $T_L$  of event no. 2, in the column labelled "Beginning Event Possible  $T_L$ ."

Notice that event no. 2 is the beginning event for activity 2-4 as well as for activity 2-5. The possible  $T_L$  value you just calculated for event no. 2 applies only to activity 2-5. It may not be smaller than that for activity 2-4. Therefore, do not star this possible value yet and do not write it in the box over event no. 2 in the network.

Panel C now looks like this. If it doesn't, correct it.

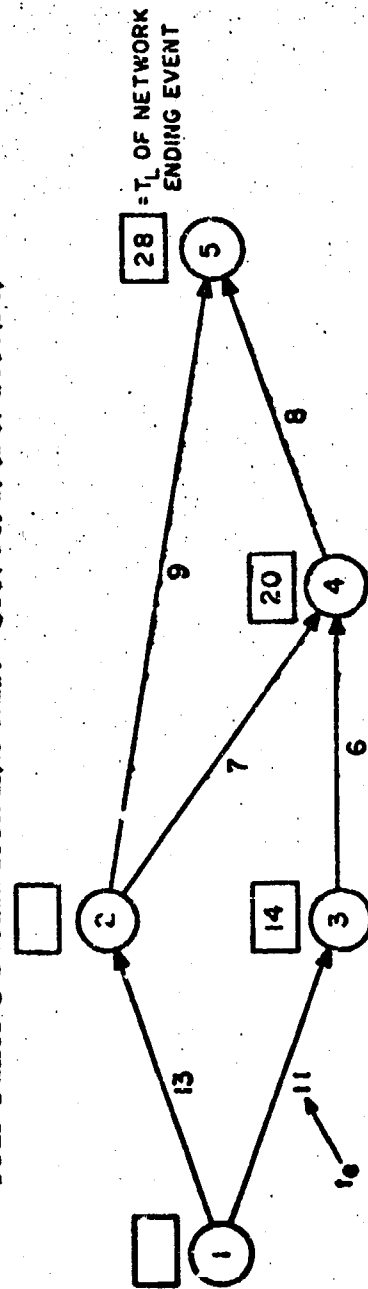


	ACTIVITY		ACTIVITY $i_a$	ENDING EVENT $T_L$	BEGINNING EVENT POSSIBLE $T_L$
	BEGIN. EVENT	ENDING EVENT			
STEP 6	1	2	13		
STEP 5	1	3	11		
STEP 4	2	4	7		
STEP 3	3	4	6		
STEP 2	2	5	9	28	19
STEP 1	4	5	8	28	20*
* = TRUE $T_L$ OF BEGINNING EVENT					

Step 3. Event no. 4 is the ending event for activity 3-4. In step 1 you found for activity 4-5, that the  $T_L$  of event no. 4 was 20. Write this value in the table of Panel C as the Ending Event  $T_L$  for activity 3-4. Subtract the  $t_e$  of activity 3-4 from this  $T_L$  and enter the result as a possible  $T_L$  of beginning event no. 3 of activity 3-4.

Notice that event no. 3 is a beginning event for only activity 3-4. Therefore the possible  $T_L$  value you just computed must be the true  $T_L$  value for event no. 3. Place a star beside this  $T_L$  value. Write this value in the box over event no. 3 in the network.

Your Panel C should look like this. Correct it if it doesn't.



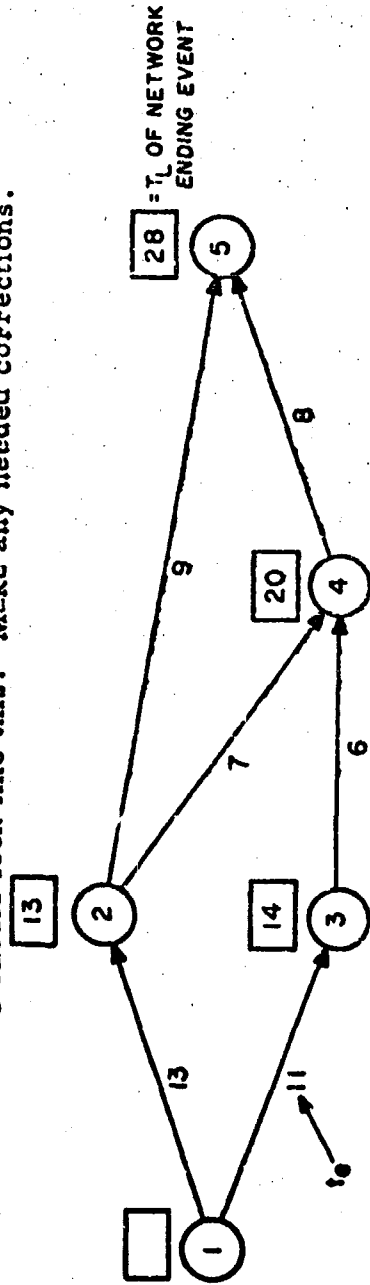
	ACTIVITY		ACTIVITY $i_0$	ENDING EVENT $T_L$	BEGINNING EVENT POSSIBLE $T_L$
	BEGIN. EVENT	ENDING EVENT			
STEP 6	1	2	13		
STEP 5	1	3	11		
STEP 4	2	4	7		
STEP 3	3	4	6	20 ←	14*
STEP 2	2	5	9	28	19
STEP 1	4	5	8	28	20*
* = TRUE $T_L$ OF BEGINNING EVENT					



Step 4. Event no. 4 is the ending event for activity 2-4 and you found that the  $T_L$  of this event is 20. Write this value as the ending event  $T_L$  of the activity 2-4. Subtract the  $t_e$  of activity 2-4 from 20 and write the result as a possible  $T_L$  of beginning event no. 2 of activity 2-4.

Look at the table. There are now two possible  $T_L$  values for beginning event no. 2. One value applies to activity 2-4, the other to activity 2-5. The smaller of these two values is the true  $T_L$  of event no. 2. Place a star beside the true value. Write the true value in the box over event no. 2 of the network.

Your Panel C should look like this. Make any needed corrections.



	ACTIVITY		ACTIVITY $t_0$	ENDING EVENT $T_L$	BEGINNING EVENT POSSIBLE $T_L$
	BEGIN. EVENT	ENDING EVENT			
STEP 6	1	2	13		
STEP 5	1	3	11		
STEP 4	2	4	7	20*	13*
STEP 3	3	4	6	20	14*
STEP 2	2	5	9	28	19*
STEP 1	4	5	8	28	20*
* = TRUE $T_L$ OF BEGINNING EVENT					

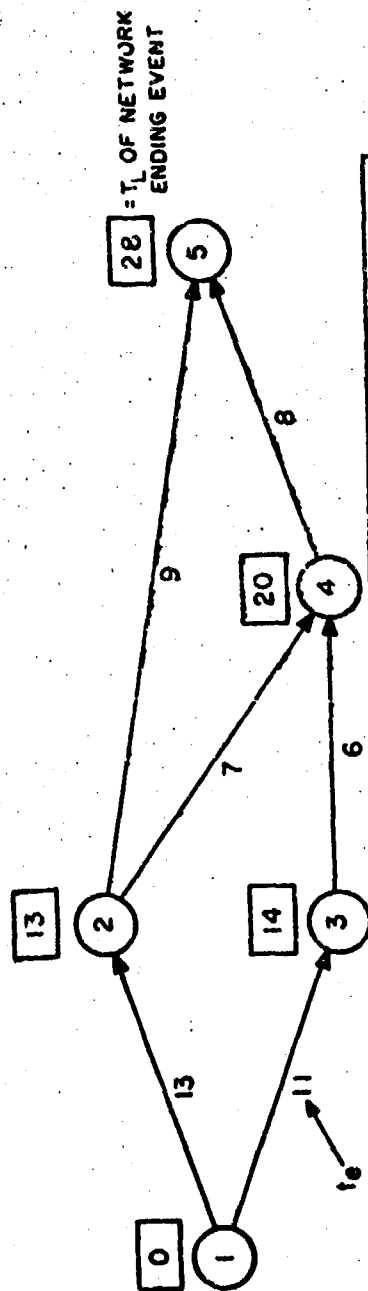
smaller than

Step 5. Write the  $T_L$  of event no. 3 which you determined in step 3, as the ending event  $T_L$  of activity 1-3. Subtract the  $t_e$  of activity 1-3 from this value. Write the result as a possible  $T_L$  for beginning event no. 1 of activity 1-3.

Step 6. Write the  $T_L$  of event no. 2 determined in Step 4, as the ending event  $T_L$  of activity 1-2. Subtract the  $t_e$  of activity 1-2 from this  $T_L$ . Write the result as a possible  $T_L$  for beginning event no. 1 of activity 1-2.

The smaller of these two possible  $T_L$  values for event no. 1 is the true  $T_L$  value. Place a star beside the true value in the table. Write the true value in the box over event no. 1 in the network.

Congratulations! You have completed Panel C. It looks like this.



	ACTIVITY		ACTIVITY $t_e$	ENDING EVENT $T_L$	BEGINNING EVENT POSSIBLE $T_L$
	BEGIN. EVENT	ENDING EVENT			
STEP 6	1	2	13	13	0*
STEP 5	1	3	11	14	3
STEP 4	2	4	7	20	13*
STEP 3	3	4	6	20	14*
STEP 2	2	5	9	28	19
STEP 1	4	5	8	28	20*
* = TRUE $T_L$ OF BEGINNING EVENT					

smaller than

Now turn to Panel D and leave it exposed to view. Start computing the  $T_L$  values of this network, and fill in the table. Use the procedures you employed for Panel C. Refer to Panel C whenever you wish.

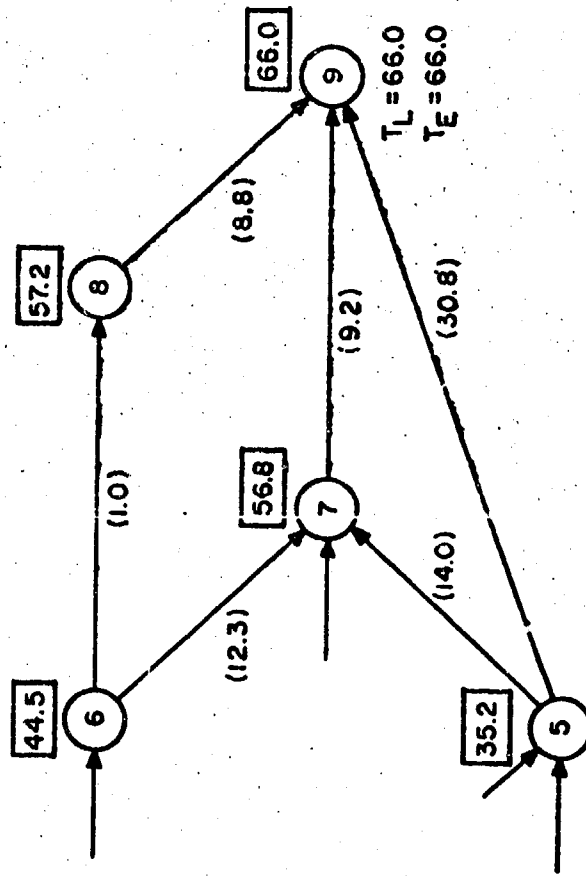
After you have completed step 7 of Panel D, turn to the next page.

Note. The network of Panel D is like that of Panel B. The  $T_E$  and  $T_L$  values of these two panels are used together later in this course.

After you have finished step 7 of Panel D, the completed part of the table should look like this. Make any necessary corrections.

Activity		Activity $t_e$	Ending Event		Beginning Event Possible $T_L$
Beginning Event	Ending Event		$T_L$	$T_L$	
Step 7	4	13.2	56.8	43.6	
Step 6	5	14.0	56.8	42.8	
Step 5	6	12.3	56.8	44.5*	
Step 4	6	1.0	57.2	56.2	
Step 3	5	30.8	66.0	35.2*	
Step 2	7	9.2	66.0	56.8*	
Step 1	8	8.8	66.0	57.2*	

After you have finished step 7 of Panel D, the completed part of the network should look like this. Make any necessary corrections.



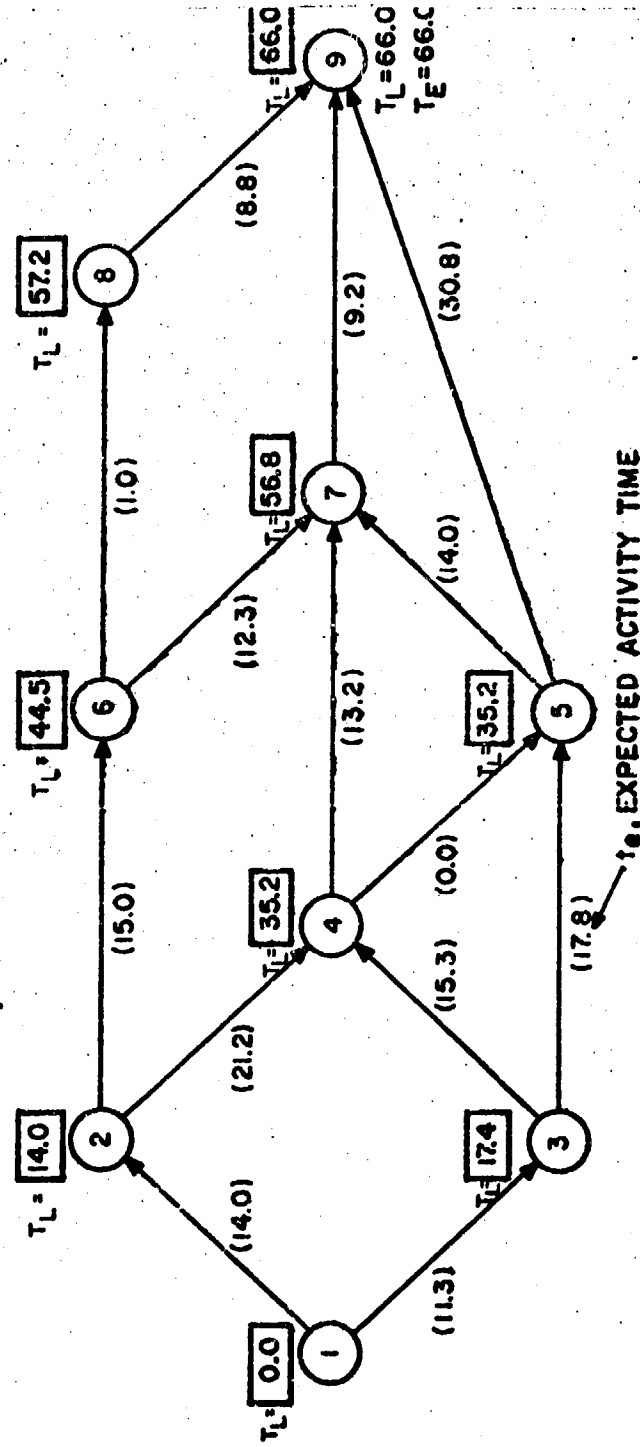
Complete the table and network of Panel D. Then turn to the next page.

After you have completed Panel D, the table should look like this. Make any necessary corrections, then turn to the next page.

Activity		Activity te	Ending Event TL	Beginning Event Possible TL
Beginning Event	Ending Event			
Step 14	1	2	14.0	0.0 *
Step 13	1	3	17.4	6.1
Step 12	2	4	35.2	14.0 *
Step 11	3	4	35.2	19.9
Step 10	3	5	35.2	17.4 *
Step 9	4	5	35.2	35.2 *
Step 8	2	6	44.5	29.5
Step 7	4	7	56.8	43.6
Step 6	5	7	56.8	42.8
Step 5	6	7	56.8	44.5 *
Step 4	6	8	57.2	56.2
Step 3	5	9	66.0	35.2 *
Step 2	7	9	66.0	56.8 *
Step 1	8	9	66.0	57.2 *



After you have completed Panel D, your network should look like this. Make corrections if necessary.



TEST NO. 1

Circle the letter identifying the phrase which appears to be most nearly correct.

1. The critical path

- a) passes through every event of a network
- b) is the least time-consuming path of activities between the network beginning event and the network ending event
- c) is the most time-consuming path of activities between the network beginning event and the network ending event
- d) includes the single most time-consuming activity of the network.

2. Always start  $T_L$  calculations at the

- a) network ending event
- b) network beginning event
- c) middle of the network
- d) either the network ending event or the network beginning event.

3.  $T_L$  is the latest time that:

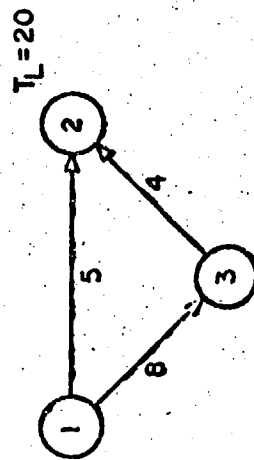
- a) an event can take place without delaying occurrence of the network ending event
- b) an activity can take place without delaying occurrence of the network ending event
- c) an event can take place after occurrence of the network ending event
- d) an activity can take place after occurrence of the network ending event.

4. For any activity,  $T_L$  of its beginning event equals:

- a)  $T_L$  of its ending event plus  $t_e$  of the activity
- b)  $T_L$  of its ending event minus  $t_e$  of the activity
- c)  $T_L$  of its ending event plus  $T_E$  of the activity
- d)  $T_L$  of its ending event minus  $T_E$  of the activity.

5.  $T_L$  for event no. 1 in the following network is:

- a) 15
- b) 16
- c) 8
- d) 12



6. In the network in question no. 5, the  $t_e$  of activity 1-3 changes from 8 to 1.  $T_L$  for event 1 is now:

- a) 16
- b) 12
- c) 15
- d) 5

If you had difficulty selecting the right answers, review Chapter 7 of Volume I and Chapter 1 of Volume II. Turn to the next page and continue the program.

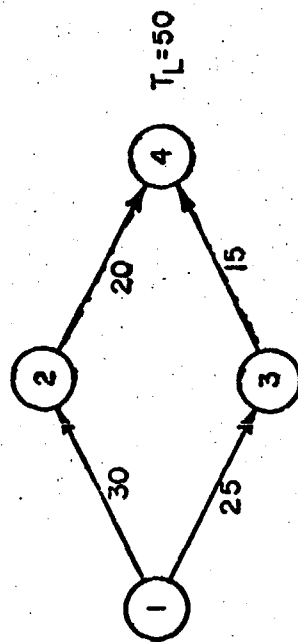
## Chapter 1

### Summary

Each event of a PERT network has a characteristic called its Latest Time or  $T_L$ . The  $T_L$  of any event is the latest time it can take place without delaying occurrence of the network ending event.  $T_L$  is calculated (backwards, so to speak) from the network ending event.

The  $T_E$  of the network ending event must be known, however, before the  $T_L$  of an event can be determined.

For any activity, the  $T_L$  of its beginning event equals the  $T_L$  of its ending event minus the  $t_e$  value of the activity. Start  $T_L$  calculations at the network ending event.



In the above network, for example, by subtracting the  $t_e$  of activity 3-4 from the  $T_L$  of event no. 4, it is possible to arrive at a  $T_L$  for event no. 3 equal to 35.

Where an event is the beginning event for more than one activity, as is the case with event no. 1, above, its  $T_L$  value must be the smallest of all possible values - in order that later events may not occur after their latest times. By performing the necessary arithmetic, two  $T_L$  values for event no. 1 will be found, only one of which is acceptable; the smaller in this case 0 (zero). Event no. 1 must, therefore, begin without delay in order that the ending event may be finished in 50 weeks.

PERTeach

Volume II

CHAPTER 2

Fundamentals of Slack

Slack is a characteristic of PERT networks that is of major importance because it focuses attention on the "time to spare" permitted for various events. Knowing the slack values which exist along different paths of activities allows management to make decisions concerning optimum use and allocation of resources.

In this chapter we are still assuming that there are no contractual or scheduling requirements that specify when the network must be completed. Under these circumstances the Latest Times of the events are calculated with respect to the  $T_E$  of the network ending event. Also the  $T_L$  of the network ending event equals its  $T_E$  value.



Each event of a PERT network has a characteristic called its Slack, S. This is the difference between the  $T_L$  of the event and the  $T_E$  of the event.

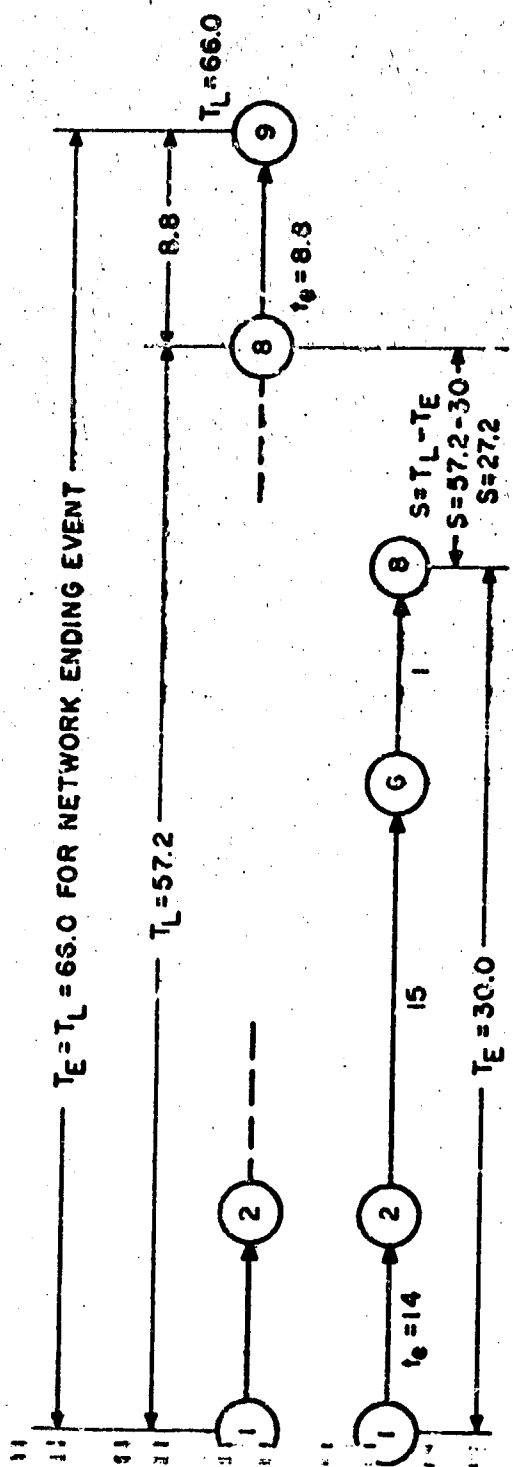
Algebraically,  $T_L - T_E = \text{---}$ .

$$T_L - T_E = \underline{S} \text{ or Slack}$$

2-60

In the network of panel D you found that the  $T_L$  of event no. 8 was 57.2 (66.0-8.8). For the same network as shown in Panel B you found that the  $T_E$  of event no. 8 was 30.0. As shown below, the difference between  $T_L$  and  $T_E$  is the Slack,  $S$ , of event no. 8. Thus  $S = T_L - T_E$  and for event no. 8.

$$S = 57.2 - 30 = 27.2$$



S is the symbol for \_\_\_\_\_.

S equals \_\_\_\_\_ minus \_\_\_\_\_.

Slack

S equals  $T_L$  minus  $T_E$  (S =  $T_L - T_E$ )

For a certain event  $T_L = 56$ , and  $T_E = 39$ .  
Therefore, S = \_\_\_\_\_.

$$S = 17$$

\_\_\_\_\_

The amount of time between the  $T_L$  of an event and the  $T_E$  of an event is called the \_\_\_\_\_ of the event.

Slack  
or  
S

\_\_\_\_\_

"Time to spare" that exists between the time when an event must occur and  
the time it is expected to occur is called \_\_\_\_\_.

Slack

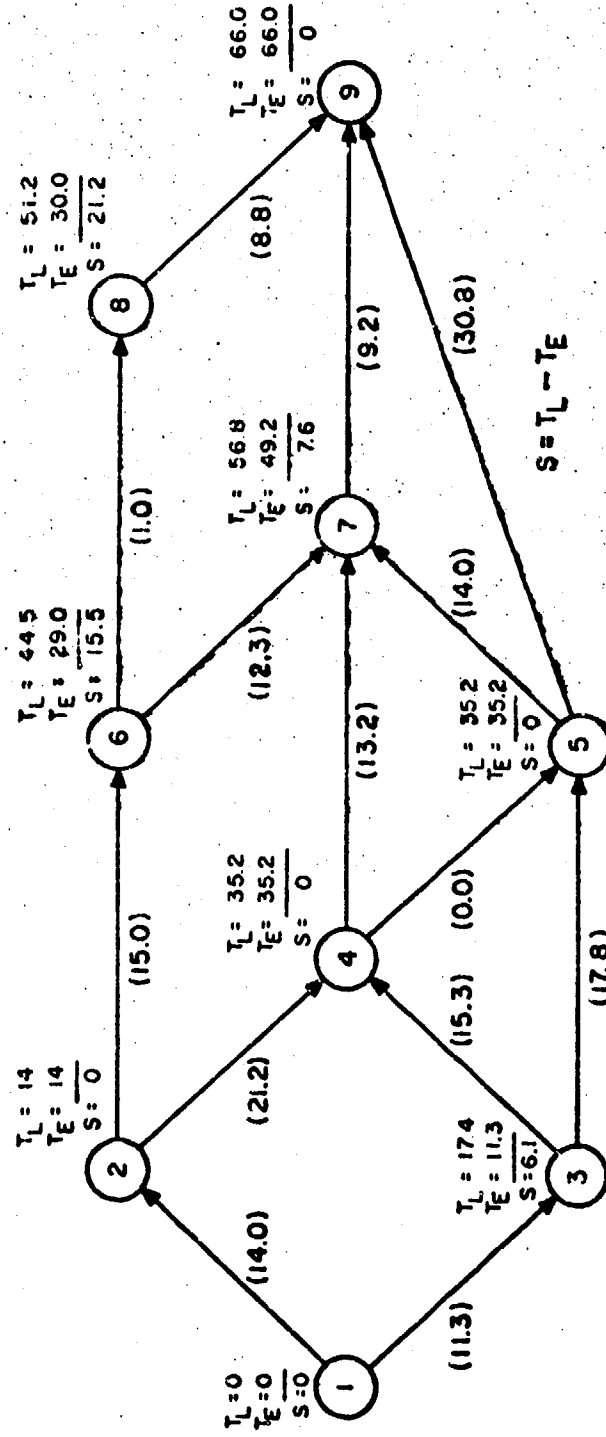
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Now open the workbook to Panel E. The network drawn here is that used for Panels B and D. Over each event is the  $T_L$  value found in Panel D and the  $T_E$  value you found in Panel B. Calculate the Slack, S, for each event. Enter your results above each event of the network.

After you have completed your computations, turn to the next page.



The network of your Panel E should look like this. Make any necessary corrections, then turn to next page.

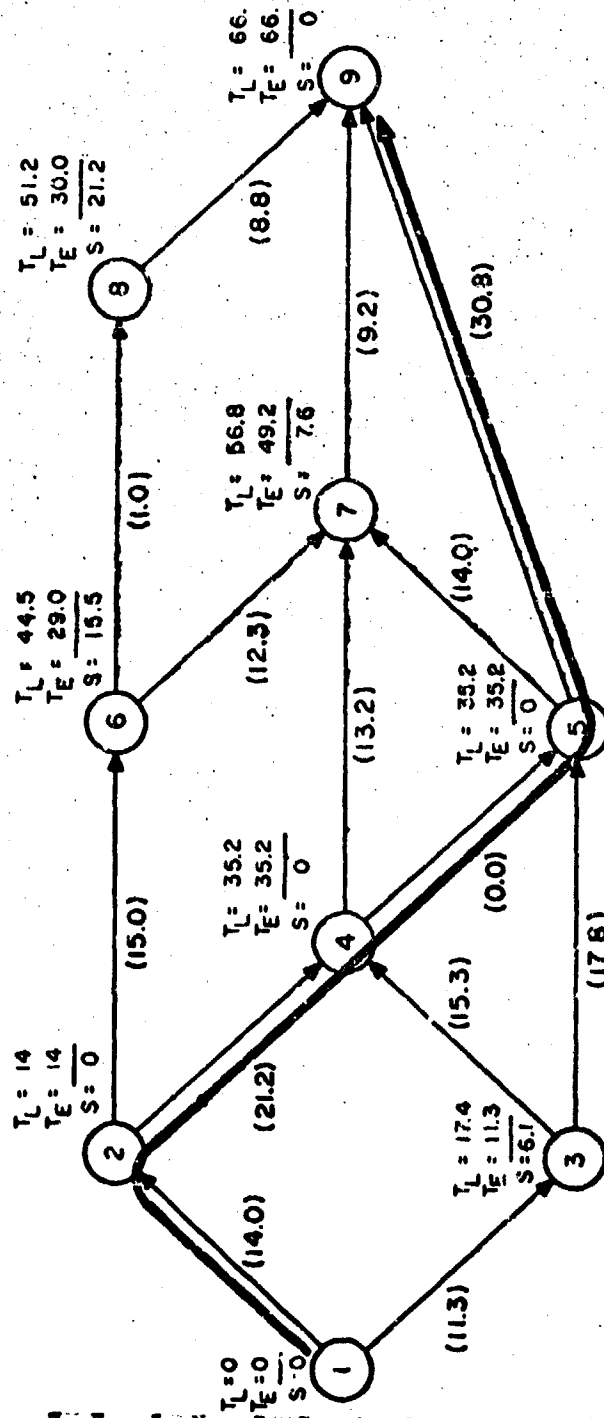


In Panel E notice that several events have Slack values equal to zero. These events are numbered 1, \_\_\_\_\_, \_\_\_\_\_, and 9.

1. 2. 4. 5. and 9

Draw a line from event no. 1 to event no. 9 over the path of activities that include the events having  $S = 0$ . Then turn to next page.

Your Panel E should now look like this. Correct it if necessary.



$S = T_L - T_E$ . If  $S = 0$ , then  $T_L - T_E = 0$  or  $T_E = T_L$ .

For an event where  $T_L = T_E$ , there is no "time to spare" between the instant when the event must occur and the instant when the event is \_\_\_\_\_ to occur.

expected (or equivalent word).

---

The critical path of the network, the path of greatest time duration through the network, determines the time required to complete it. Any delay in occurrence of events on the critical path will delay completion of the network. Therefore, these events have minimum "time to spare"; that is, their Slack values must be the minimum values of the network.

In Panel E, the activity path 1-2-4-5-9 includes all those events having zero Slack. Since zero is the minimum Slack of this network, the activity path 1-2-4-5-9 should be the \_\_\_\_\_ path of the network.

critical

Panel B uses the same network that is shown in Panel E.

As you will remember, in Panel B you found that the critical path, the activity path of greatest time through the network, was path 1-2-4-5-9.

In Panel E you found that the Path 1-2-4-5-9 includes all those events having minimum (zero) Slack.

Thus, we see that the critical path of this network actually is the activity path whose events have \_\_\_\_\_ Slack.

zero or minimum

2-74



TEST NO. 2

Circle the letter identifying the phrase which appears to be most nearly correct.

1. Slack is:

- a) The amount of time remaining at each event to complete the project.
- b) The amount of time an event can be delayed without delaying the Network Ending Event.
- c) The expired time from the Network Beginning Event.
- d) None of the above.

2. The formula for slack is:

- a)  $TL + TE$
- b)  $TE - TL$
- c)  $TL - TE$
- d)  $te - TL$

3. What is the slack for an event if the event  $T_L$  is 20 and the event  $T_E$  is 5?

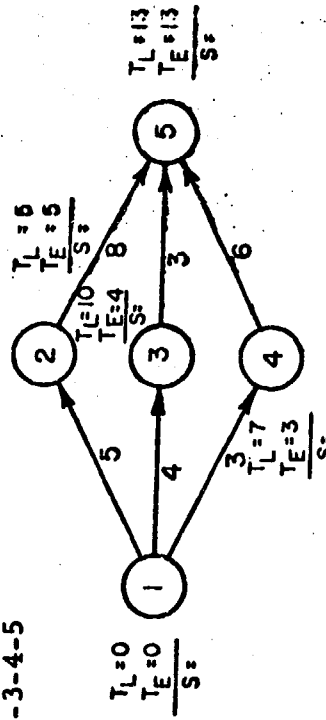
- a) 20
- b) 15
- c) 10
- d) 5

4. Assuming no schedule date for the network ending event, the slack time of the events in the critical path are always:

- a) maximum values of the network.
- b) zero.
- c) different from each other.
- d) are always unequal.

5. In the network below, the critical path passes through events:

- a) 1-3-5
- b) 1-4-5
- c) 1-2-5
- d) 1-3-4-5



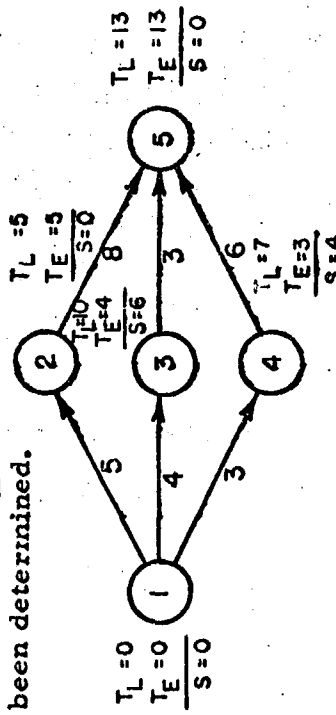
If you had difficulty selecting the correct answers, review Chapter 2.  
Turn to the next page.

## Chapter 2

### Summary

Time to spare that exists between the time when an event must occur ( $T_L$ ) and the time it is expected to occur ( $T_E$ ) is called Slack( $S$ ). Knowing which events have time to spare, or slack, is crucial to the manager who makes decisions concerning optimum allocation of resources.

Algebraically, slack or  $S = T_L - T_E$ . In the network below, for example, all slack values have been determined.



Notice that activity path 1-2-5 has the minimum slack (zero) in this network. This path with least time to spare is, conversely, the path of greatest time through the network - in other words, the critical path.

PER Teach

Volume II

CHAPTER 3

Schedule Date

In chapters 1 and 2 we assumed there were no requirements specifying the calendar date when a network was to be completed. In that case, the latest time,  $T_L$ , of the network ending event equals its Accumulated Expected Time,  $T_E$ .

The  $T_E$  of the network ending event can be considered the time expected to be required to accomplish the complete network. The  $T_L$  of the network ending event can be considered the time allowed to accomplish the network.

A Schedule Date,  $T_S$ , is a predetermined calendar date by which a given event is scheduled to occur. Usually Schedule Dates are applied only to network beginning and ending events.

In this chapter we assume there is a Schedule Date,  $T_S$ , specifying a calendar date by which the network is to be completed and another  $T_S$  specifying the date when it is to be started. The elapsed time corresponding to the difference between these two dates is the time allowed to accomplish the network. The  $T_L$  of the network ending event now equals this elapsed time. Setting dates on a project does not, however, modify the time required to accomplish it. Therefore the  $T_E$  of the network ending event is not affected by  $T_S$  values.

A predetermined calendar date by which an event is scheduled to occur is called the Schedule Date of the event and is symbolized by TS.

An example of a Schedule Date is 19 September 1962. The calendar date, 23 April 1975, is another example of a \_\_\_\_\_.

Schedule Date

\_\_\_\_\_

$T_S$  is the symbol for \_\_\_\_\_.



**Schedule Date**

\_\_\_\_\_

A predetermined calendar date by which an event is scheduled to occur is called the Schedule Date of the event and is symbolized by \_\_\_\_\_.

$T_S$

---

In PERT terminology, Schedule Date,  $T_S$ , is a pre-determined calendar date.

The Accumulated Expected Time,  $T_E$ , of an event or its Latest Time,  $T_L$ , are not calendar dates. Instead they are elapsed times.

To compute  $T_E$  we start at the network beginning event. To compute  $T_L$  we start at the network ending event. However, both  $T_E$  and  $T_L$  are expressed as time (in weeks) that has elapsed since occurrence of the network beginning event.

To use Schedule Date,  $T_S$ , in conjunction with Accumulated Expected Time,  $T_E$ , or Latest Time,  $T_L$ , we must change the calendar date of  $T_S$  into an \_\_\_\_\_ measured from occurrence of the network beginning event.

elapsed time

The  $T_E$  and  $T_L$  of a given event are elapsed times measured from occurrence of the network beginning event. They are expressed in units and tenths of a 7-day week.

The elapsed time corresponding to the  $T_S$  of an event is also expressed in units and tenths of a \_\_\_\_\_.

7-day week

---

If  $TS_2$  is the Schedule Date of a given event and  $TS_1$  is the Schedule Date of the network beginning event, then the elapsed time corresponding to  $TS_2$  is expressed as the number and tenths of 7-day weeks between  $TS_2$  and  $TS_1$ .

To determine the number of weeks, find the number of days between  $TS_2$  and  $TS_1$ , then divide the result by seven.

Suppose  $TS_1$  is 15 May 1962 and  $TS_2$  is 11 October 1962. There are 31 days in May, 30 in June, 31 each in July and August, and 30 in September. The elapsed time between the above two calendar dates is 149 days. Dividing 149 by 7 gives 21.3 weeks as the elapsed time between 15 May 1962 and 11 October 1962.

Thirty days hath September,

April, June and November,

All the rest have 31,

Except February which has 28.

(Yes, we know - 29 in Leap Year.)

The network beginning event of a certain project is scheduled to occur on 1 July 1962 and the  $T_S$  of the network ending event is 15 February 1963. Expressed in units and tenths of a 7-day week, the elapsed time corresponding to the  $T_S$  of the network ending event is \_\_\_\_ weeks.

32.85 weeks

Suppose a network beginning event has a Schedule Date,  $TS$ , of 1 January 1962 and the network ending event has a  $TS$  of 1 January 1963. Then the elapsed time between these two  $TS$  values is 52.0 weeks. The network ending event is allowed to occur at any time within these limits but the Latest Time at which it can occur within these limits must be 52.0 weeks after the network beginning event takes place. Thus the elapsed time between the  $TS$  values of the network beginning and ending events equals the \_\_\_\_\_,  $T_L$ , of the network \_\_\_\_\_ event.

latest time

ending

2-89

Suppose that a project represented by a PERT network must be started on 1 January 1962 and is scheduled to end on 8 January 1963. These are the  $T_S$  dates of the network beginning and ending events respectively. The elapsed time allowed for this project is 53 weeks. The Latest Time,  $T_L$ , of the network ending event must equal 53 weeks. If this event takes place after more than 53 weeks, it will occur after 8 January 1963. By summing the  $t_e$  values of the activity paths we find that the Accumulated Expected Time,  $T_E$ , of the network ending event is 48 weeks. This is the elapsed time expected to be required to accomplish the entire network. It is not related to the  $T_L$  or  $T_S$  values.

Later, through contract revisions, the Schedule Date of the network ending event is changed from 8 January 1963 to 22 January 1963.

The Latest Time,  $T_L$ , of the network ending event is now \_\_\_\_\_ weeks.

The Accumulated Expected Time,  $T_E$ , of the network ending event is now \_\_\_\_\_ weeks.



$T_L = 55 \text{ weeks}$

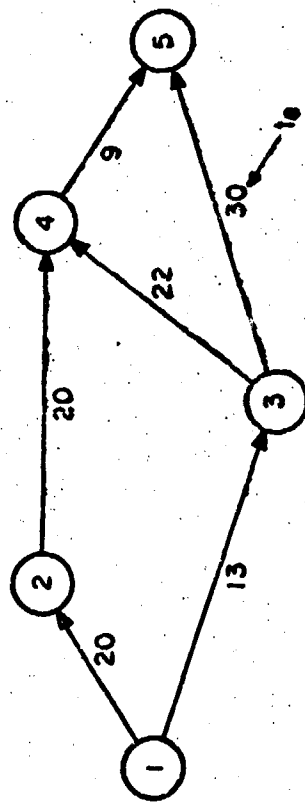
$T_E = 48 \text{ weeks}$

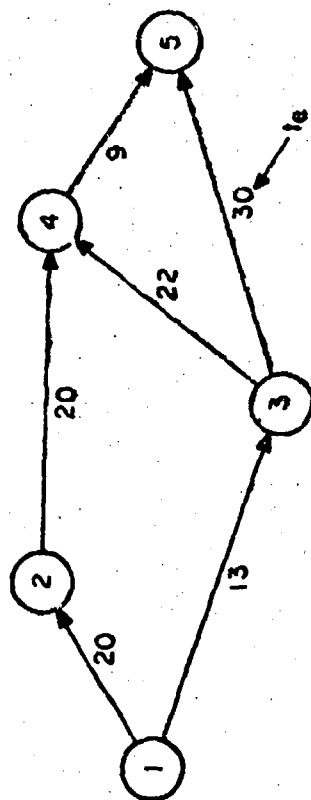
In the network below the  $T_S$  of the network beginning event is 1 January 1962 and the  $T_S$  of the network ending event is 24 December 1962.

The Latest Time,  $T_L$ , of the network ending event no. 5 is \_\_\_\_ weeks.

The Accumulated Expected Time,  $T_E$ , of network ending event no. 5 is \_\_\_\_ weeks.

For the network ending event, Slack,  $S = T_L - T_E =$  \_\_\_\_ weeks.





$T_L = 51$  weeks

$T_E = 49$  weeks

$S = 2$  weeks

The  $T_E$  of a network ending event is 31 weeks.

The elapsed time corresponding to the  $T_S$  of the network ending event is 39 weeks.

The  $T_L$  of the network ending event is \_\_\_\_ weeks.

The  $S$  of the network ending event is \_\_\_\_ weeks.

$T_L = 39$  weeks

$S = 8$  weeks

---

Although Schedule Date,  $T_S$ , is commonly applied to network beginning and ending events, it should be remembered that  $T_S$  can apply to any event of a PERT network. In this case the elapsed time corresponding to the  $T_S$  of the event becomes the  $T_L$  of the event. This  $T_L$  value supersedes any other value that may be calculated.

Suppose the  $T_L$  of an event within a network is 27.0 weeks but that later a Schedule Date is imposed on this event. Assume the elapsed time corresponding to this date is 25.0 weeks after occurrence of the network beginning event. Then the  $T_L$  of this event becomes \_\_\_\_\_ weeks.

25 weeks

---

We calculate that the  $T_L$  of a given event is 17 weeks. Soon after the program starts, however, a Schedule Date is imposed on this event and the elapsed time corresponding to this date is 14 weeks. Then to calculate the Latest Times of events which occur before this event and which are on the same path of activities we use a  $T_L$  value of \_\_\_\_ weeks for the given event.

[illegible][illegible]

TEST NO. 3

Circle the letter identifying the phrase which appears to be most nearly correct.

1. T<sub>S</sub> stands for:

- a) slack time.
- b) summary time.
- c) schedule date.
- d) estimated time.

2. T<sub>S1</sub> is 1 January 1962. T<sub>S2</sub> is 11 February 1962. Elapsed time between the two, expressed in units and tenths of a week, is:

- a) 4.5
- b) 6.0
- c) 8.4
- d) 10.0



3.  $TS_1$  is 1 July 1962,  $TS_2$  is 31 May 1963. Elapsed time, expressed in units and tenths of a week, is:

- a) 47.9
- b) 67.2
- c) 36.0
- d) 45.7

4. A certain network ending event is given a  $TS$  value. The corresponding elapsed time is used in calculating:

- a) Accumulated Expected Time
- b) Estimated Time
- c) Expected Time
- d) Slack Time

5.  $TS$  can be applied to:

- a) only an activity.
- b) either activities or events.
- c) only a network ending event.
- d) any event in a network.

If you had difficulty in selecting the correct answers, review Chapter 3. Now turn the page and continue the program.

## Chapter 3

### Summary

Both  $T_E$  and  $T_L$  are elapsed times, not calendar dates.  $T_S$  or the Schedule Date, on the other hand, is a predetermined calendar date by which an event is scheduled to occur.

To use  $T_S$  in conjunction with  $T_E$  or  $T_L$ ,  $T_S$  must be changed into an elapsed time measured from occurrence of the network beginning event.  $T_S$  is expressed in units and tenths of seven-day weeks. If a  $T_S$  is assigned to an event, the corresponding elapsed time becomes the  $T_L$  of the event.

Suppose, for example, that  $T_{S1}$  (the Schedule Date of the network beginning event) is 15 May 1962, while  $T_{S2}$  (the Schedule Date of a particular given event) is 11 October 1962. The elapsed time between these two dates is 149 days divided by 7 or 21.3 weeks.

PERTeach

Volume II

CHAPTER 4

Positive and Negative Slack

2-101

Slack is a PERT characteristic of major importance to a manager for it provides the replanning or allocation information on which he can base rational decisions. Slack takes account of other PERT concepts such as  $T_E$ ,  $T_L$ , and  $T_S$  to point out the network areas having excesses or deficiencies of time.

For any event, Slack,  $S$ , equals its Latest Time,  $T_L$ , minus its Accumulated Expected Time,  $T_E$ . That is,  $S = T_L - T_E$ .

The  $T_E$  of an event represents the time expected to elapse before the event will take place. It is not dependent on  $T_L$  or the Schedule Date,  $T_S$ , the calendar date by which the event is to occur. On the other hand, the  $T_L$  of an event equals the elapsed time corresponding to the  $T_S$  of the event,  $T_L$  is the latest time the event can occur without delaying scheduled completion of the network. It is the time allowed for occurrence of the event. Obviously, the Slack of an event will be zero, positive, or negative, depending on whether  $T_L$  is equal to, greater than, or less than  $T_E$ .

If  $T_L$  and  $T_E$  are those of the network ending event, the value of  $S$  applies not only to this event but to all events on the critical path.

In this chapter we will explore the relation between Schedule Date of the network ending event and Slack, and we will consider some characteristics of Slack.

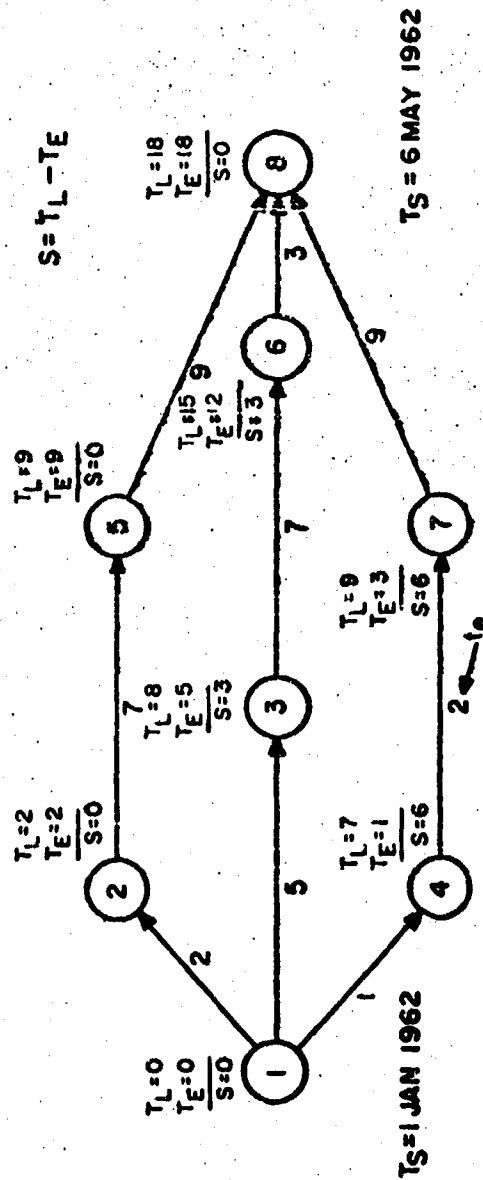
Open the workbook to Panel F. In this panel the same network is repeated three times, as Case 1, Case 2, and Case 3. We will work first only with Case 1.

Slack,  $S = T_L - T_E$ . When the  $T_L$  of a network ending event equals its  $T_E$  value,  $S = 0$  for all events on the critical path. This common Slack value is smaller than that for any event not on the critical path.

To prove this, consider Case 1 of Panel F. Here the network is to be started on 1 January 1962 and is to be completed by 6 May 1962, 18 weeks later. This elapsed time becomes the  $T_L$  of the network ending event. The greatest sum of activity  $t_e$  values along any path of activities between network beginning and ending events is 18 weeks, the  $T_E$  of the network ending event. Thus,  $T_L = T_E$  for the network ending event, and its Slack value is zero.

Compute the Slack,  $S$ , for each event of Case 1.

Case 1 of your Panel F should look like this. Correct it if necessary.





In case 1,  $T_L$  of the network ending event equals its  $T_E$  value.

In the network of Case 1 the critical path includes events numbered 1, \_\_\_\_\_, \_\_\_\_\_.

8. Draw a line over this activity path.

When the  $T_L$  of the network ending event equals its  $T_E$  value, the Slack of each event on the critical path equals \_\_\_\_\_.

The Slack value of events on the critical path is \_\_\_\_\_ (smaller / greater) than those of events that are not on the critical path.

Events numbered 1, 2, 5, 8.

zero or 0

smaller

\_\_\_\_\_

Refer to Case 1, Panel F.

If Schedule Dates for the beginning and ending events of a network result in a Slack of zero along the critical path, there is \_\_\_\_\_ (not/just) enough time allowed to complete the network on schedule.

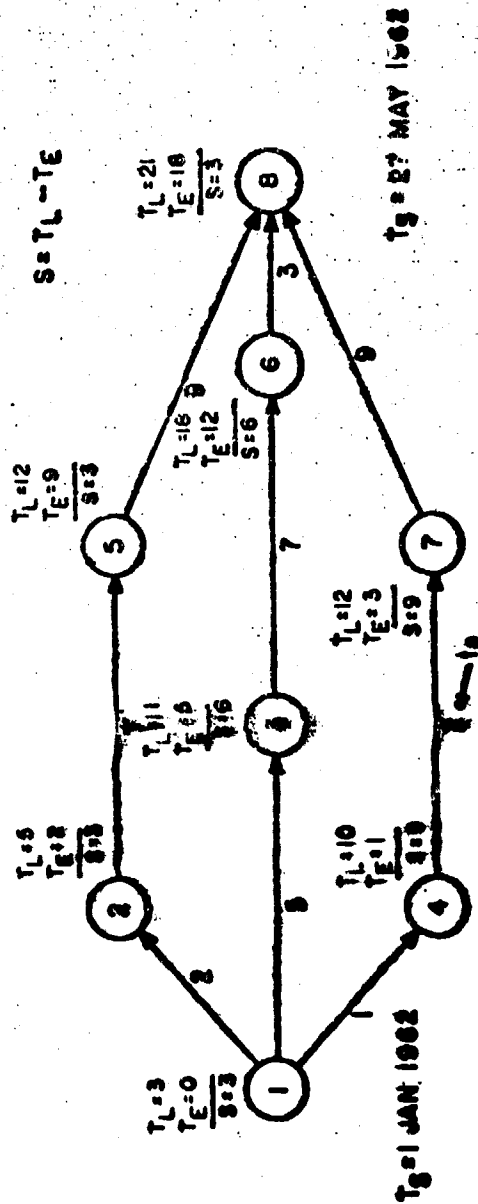
just

Slack,  $S = T_L - T_E$ . When the  $T_L$  of a network ending event is greater than its  $T_E$  value, the Slack values of events on the critical path are positive and equal to each other. This common Slack value is smaller than that of any event not on the critical path.

To prove this, consider Case 2 of Panel F. Here the network is to be started on 1 January 1962 and is to be completed by 27 May 1962, 21 weeks later. This elapsed time becomes the  $T_L$  of the network ending event. The  $T_E$  of the network ending event, 13 weeks, remains unchanged. Thus, in Case 2,  $T_L$  is greater than  $T_E$  for the network ending event.

Compute the Slack,  $S$ , for each event of Case 2. Write your answers in the spaces above the corresponding events of the network.

Case 2 of your Panel F should look like this. Correct it if necessary.



0-110

In Case 2 the TL of the network ending event is greater than its TE value.

For the network of Case 2, the critical path includes events numbered 1, \_\_\_\_\_.

8. Draw a line over this activity path.

When the TL of the network ending event is greater than its TE value, the Slack values of events on the critical path are \_\_\_\_\_ (positive/negative) and are \_\_\_\_\_ (different from/equal to) each other.

The Slack values of events on the critical path are \_\_\_\_\_ (larger/smaller) than those of events not on the critical path.

Events numbered 1, 2, 5, 8.

positive

equal to

smaller

\_\_\_\_\_

Refer to Case 2, Panel F.

If Schedule Dates for the beginning and ending events of a network result in a positive Slack along the critical path, there is \_\_\_\_\_ (more/less) than enough time allowed to complete the network on schedule.

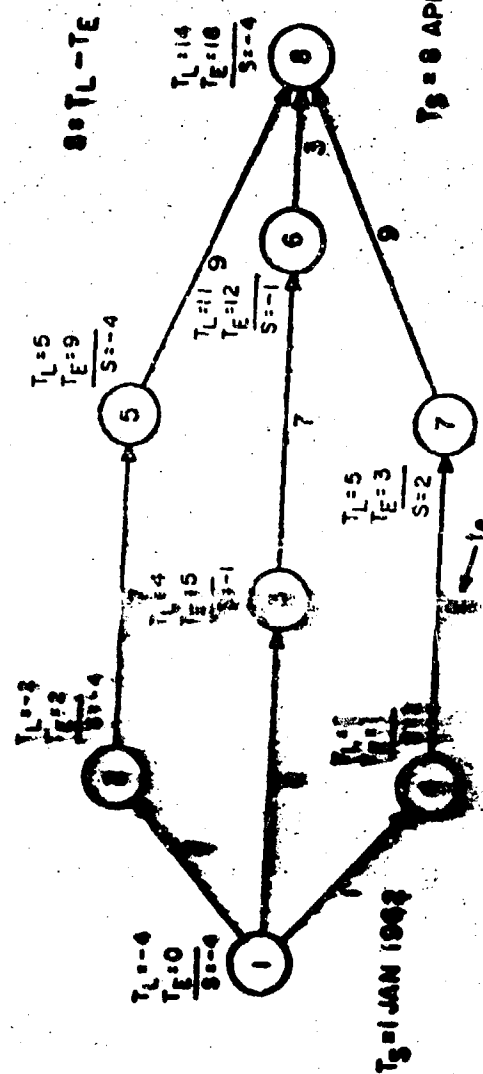
more

Slack,  $S$ ,  $= T_L - T_E$ . If the  $T_L$  of a network is less than its  $T_E$  value, the Slack values of events on the critical path are negative and equal to each other. The Slack values of other events may be either positive or negative but the common negative value of those on the critical path is a greater negative number than that of any other event.

Consider Case 3 of Panel F. Here the network is to be started on 1 January 1962 and is to be completed by 8 April 1962. The elapsed time between these two dates, 14 weeks, becomes the  $T_L$  of the network ending event. The  $T_E$  of the network ending event, 14 weeks, remains unchanged. Thus, in Case 3, the  $T_L$  of the network ending event is less than its  $T_E$  value.

Compute the Slack for each event of Case 3. Write your answers above the corresponding events of the network.

Case 3 of your Panel F should look like this. Correct it if necessary.





In Case 3 of Paper V, the  $T_L$  of the network ending event is less than its  $T_E$  value.

For the network of Case 3, events on the critical path are numbered 1, \_\_\_\_\_.

8. Draw a line over this activity path.

If the  $T_L$  of the network ending event is less than its  $T_E$  value, the Slack values of events on the critical path are \_\_\_\_\_ (positive/negative) and are \_\_\_\_\_ from/equal to each other.

The Slack values of events on the critical path are more \_\_\_\_\_ (positive/negative) than that of any other event.

Events numbered 1, 2, 5, 8.

negative

equal to

negative

Refer to Case 1, Panel D

If Schedule Dates for the beginning and ending events of a network result in a negative Slack along the critical path, there is \_\_\_\_\_ (more/less) than enough time allowed to complete the program on schedule.

less

If the Slack of the critical path is negative, not enough time is being allowed to complete the network. Under these circumstances, or wherever the Slack of the critical path is unacceptable, it may be possible to replan the network in one or more of the following ways.

1. Change an activity path composed of series-connected activities into a series-parallel group of paths.
2. Change the resources applied to the critical path.
3. Change the scope of various activities.

In Case 1 of Panel F, the Slack of the network beginning event, no. 1 is zero.  
This means that the event must occur \_\_\_\_\_ (on/after) 1 January 1962 if the  
network ending event, no. 8, is to take place by 6 May 1962.

qn

In Case 2, the slack of the network beginning event, no. 1, is 3 weeks. This means that the event may occur 3 weeks (before/after) 1 January 1962 and still have the network ending event occur by 27 May 1962.

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before

Look at the critical paths outlined in the three cases of Panel F. From these sketches we conclude that changing the  $T_L$  or  $T_S$  of the network ending event \_\_\_\_\_ (always/never) changes the critical path.

never

The events on any one activity path have the same value of Slack until that path intersects another.

For example, in Case 1 of Panel F, the Slack of events nos. 3 and 6, on activity path 1-3-6-8, is 3 weeks.

Similarly events nos. 4 and 7, on activity path 1-4-7-8, have a common Slack value of \_\_\_\_\_ weeks.



SIX or 6 weeks

---

The Slack of an event that is at the intersection of two or more activity paths is equal to the smallest possible value.

For example, event no. 1 of Case 2, Panel F, is at the intersection of three activity paths along which Slack values are 3, 6, and 9 weeks, respectively. The Slack of event no. 1 is the smallest of these, at 3 weeks. This result is not dependent on the fact that event no. 1 is a network beginning event.

The Slack along the critical path is the \_\_\_\_\_ (minimum/maximum)  
possible in the network.

minimum

The Slack value along the critical path of a network is a measure of the "time to spare" for completion of the network. If two activity paths give the same value of  $T_E$  for the network ending event, both are called \_\_\_\_\_ paths.

critical

---

The critical path of a network has minimum Slack and therefore is of major importance to a manager who bases decisions on PERT information. However, other activity paths having Slack values somewhat greater than that of the critical path must also be considered. Any network changes that decrease the elapsed time represented by the critical path or which increase the time represented by other activity paths may transform a non-critical path into a critical one.

In Case 3 of Panel F,  $S = -4$  for the critical path 1-2-5-8. Suppose replanning reduces the  $t_e$  of activity 4-5 from 7 weeks to 3 weeks. This does not change the  $T_L$  of the network ending event because its  $T_S$  remains the same.

The  $T_E$  of the network ending event is now \_\_\_\_\_ weeks.

The critical path is now 1-\_\_\_\_\_, \_\_\_\_\_-8.

The Slack along the critical path is now \_\_\_\_\_ weeks.

The Slack of path 1-2-5-8 is now \_\_\_\_\_ weeks.

The  $T_E$  of the network ending event is now 15 weeks.

The critical path is now 1-3-6-8.

The Slack along the critical path is -1 week.

The Slack of path 1-2-5-8 is now zero weeks.

If your answer was incorrect:

- a) check your computation.
- b) remember that the critical path may change if  $t_e$  values are altered.

TEST NO. 4

Circle the letter identifying the phrase which appears to be most nearly correct.

1. When the  $T_L$  value of a network ending event is greater than its  $T_E$  value, the slack value of events on the critical path are:
  - a) positive and unequal
  - b) negative and equal
  - c) positive and equal
  - d) negative and unequal
2. If a critical path has negative slack, there is:
  - a) insufficient time to complete the network on schedule
  - b) more than enough time to complete the network on schedule
  - c) exactly enough time to complete the network on schedule
  - d) none of the above

3. The slack values of two activity paths are the same and this value is the minimum in the network. These paths are:

- a) non-critical paths
- b) critical paths
- c) slack paths
- d) none of the above

4.  $T_S$  of a network ending event is 4 March 1963. The slack for this event is positive and equal to 3.0 weeks. Therefore, the event must occur no later than:

- a) 4 March 1963
- b) 25 March 1963
- c) 11 February 1963
- d) 7 March 1963

If you had difficulty selecting the right answers, review Chapter 4.



## Chapter 4

### Summary

The slack of an event will be zero, positive, or negative depending upon whether  $T_L$  is equal to, greater than, or less than  $T_E$ .

The slack value of events on the critical path is smaller than those of events not on the critical path.

If the slack of the critical path is negative, not enough time is being allowed to complete the network. Altering the  $T_L$  or  $T_S$  of the network ending event never changes the critical path.

The manager must pay attention to paths other than the critical one. Any network changes that decrease the elapsed time represented by the critical path or which increase the time represented by other activity paths may transform a non-critical path into a critical one.

Turn this page, turn the book around and continue the program on page 2-132.

PERTeach

Volume II

CHAPTER 5

PERT Probability

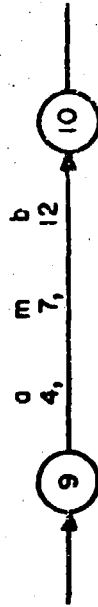
2-132

In previous chapters we have been concerned with those aspects of PERT that tell a manager how long it should take to complete a project and whether there appears to be sufficient time to meet scheduled completion dates. However, all elapsed time information provided by a network contains inherent uncertainties. This is so because such information is primarily based on the three time estimates (Pessimistic Time, b, Most Likely Time, m, and Optimistic Time, a) required for each activity.

Such estimates involve varying degrees of uncertainty. Consequently, the Expected Activity Times ( $t_E$ ) of the network activities, as well as the Accumulated Expected Times, ( $T_E$ ) of the events, must also be uncertain.

This chapter is concerned with statistical measures of these uncertainties and the resulting probabilities that events, particularly the network ending event, will occur at the times calculated. To a manager this statistical information can be of interest. It tells him the odds of completing a group of activities or an entire network within the calculated expected time, and provides a measure of the risks involved in trying to meet a given schedule.

You will remember that for each activity, three time estimates are required. These are Optimistic Time, a, Most Likely Time, m, and Pessimistic Time, b. They are arranged over an activity as shown below.



The time spread or span between Pessimistic Time, b, and Optimistic Time, a, is expressed by the quantity  $(b - a)/6$ . This quantity is called Activity Standard Deviation. The symbol for Activity Standard Deviation is the Greek letter "sigma", written  $\sigma$ .

Symbolically  $\sigma = (b - a)/6$ .

For activity 9-10 above:  $\sigma$  equals  $(b - a)/6 = (12 - 4)/6 = 1.3$ .

If m is changed from 7 to any other value, say 5 or 9, the Activity Standard Deviation,  $\sigma$ , would still be  $(12 - 4)/6$  or 1.3.

The value of Most Likely Time, m, \_\_\_\_\_ (always/never) affects the value of Activity Standard Deviation,  $\sigma$ .

never

\_\_\_\_\_

The symbol for Activity Standard Deviation is \_\_\_\_\_.

$(b - a)/6 =$  \_\_\_\_\_

The symbol for Activity Standard Deviation is  $\sigma$ .

$$(b - a)/6 = \sigma$$

\_\_\_\_\_

$\sigma$  is the symbol for \_\_\_\_\_.

$\sigma =$  ( \_\_\_\_\_ )  
Letter Minus Sign Letter Number

$\sigma$  is the symbol for Activity Standard Deviation.

$$\frac{b - a}{3}$$

For our purposes, the value of  $\sigma$  need be accurate to only the first decimal place. Thus, although we may calculate that  $\sigma = 2.33$ , we use the value 2.3.

Make calculations to two decimal places. If the digit in the second decimal place (the digit 8 in 1.28) is equal to or greater than 5, increase the digit in the first decimal place (the digit 2 in 1.28) by one and then discard the digit in the second decimal place. Thus 1.28 becomes 1.3.

If the digit in the second decimal place is less than 5, discard it without changing the rest of the number. Thus 1.24 becomes 1.2.

According to the above rules:

$$2.16 = \underline{\hspace{1cm}}$$

$$2.33 = \underline{\hspace{1cm}}$$

$$2.66 = \underline{\hspace{1cm}}$$

$$2.83 = \underline{\hspace{1cm}}$$

$$9.85 = \underline{\hspace{1cm}}$$

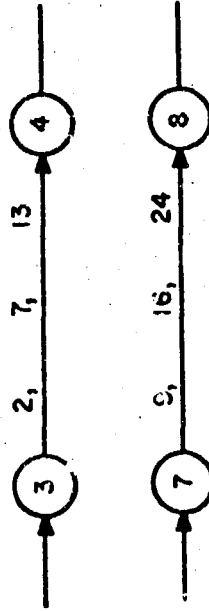
$$9.95 = \underline{\hspace{1cm}}$$

2.16 = 2.2   2.33 = 2.3   2.66 = 2.7  
 2.83 = 2.8   9.85 = 9.9   9.95 = 10.0

\_\_\_\_\_

For activity 3-4,  $\sigma =$  \_\_\_\_\_.

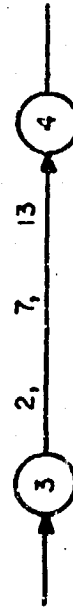
For activity 7-8,  $\sigma =$  \_\_\_\_\_.





For activity 3-4,  $\sigma = (13 - 2)/6 = 11/6$  or 1.8.

For activity 7-8,  $\tau = (24 - 9)/6 = 15/6$  or 2.5.



Activity Standard Deviation,  $\sigma$ , is a measure of the spread between the Pessimistic and Optimistic Time estimates,  $b$  and  $a$ . However, the position of Most Likely Time,  $m$ , can vary appreciably between  $a$  and  $b$  among several activities. Consequently, Activity Standard Deviation,  $\sigma$ , has little useful significance as far as the activities themselves are concerned.

However, if we consider several activities connected in series, the different positions of the  $m$  values tend to cancel each other. In this case, the square root of the sum of the squares of Activity Standard Deviation gives the Standard Deviation of the event which terminates the series of activities. This is called Event Standard Deviation,  $\sigma_{TE}$ .

Because the Accumulated Expected Time,  $T_E$ , of an event is basically the result of a series of time estimates, rather than of a group of known values of time, there is some uncertainty associated with the calculated  $T_E$  value. Event Standard Deviation,  $\sigma_{TE}$ , is a valid statistical measure of this uncertainty, particularly if the event in question terminates a series of ten or more activities.

Knowing the Event Standard Deviation,  $\sigma_{TE}$ , of an event and its  $T_E$  value permits us to calculate the probability of accomplishing the event by any given Schedule Date,  $T_S$ .

$\sigma_{TF}$  is the symbol for

2-141

Event Standard Deviation

\_\_\_\_\_

The symbol for Event Standard Deviation is \_\_\_\_\_.

$\sigma_{TE}$

Activity time estimates, a, m, and b, are expressed in units and tenths of a 7-day week. Expected Activity Time ( $t_e$ ) as well as Accumulated Expected Time ( $TE$ ) and Latest Time ( $T_L$ ) for events are also expressed in units and tenths of a 7-day week. Activity Standard Deviation ( $\sigma$ ) and Event Standard Deviation ( $\sigma_{TE}$ ) are also expressed as \_\_\_\_\_ and \_\_\_\_\_ of a \_\_\_\_\_.

Activity Standard Deviation ( $\sigma$ ) and Event Standard Deviation ( $\sigma_{TE}$ ) are also expressed as units and tenths of a 7-day week.

For several activities connected in series, the square root of the sum of the squares of the Activity Standard Deviations,  $\sigma$ , equals the Event Standard Deviation,  $\sigma_{TE}$ , of the event terminating the series.

Expressed symbolically for the series of activities below,

$$\sqrt{(\sigma_{1-2})^2 + (\sigma_{2-3})^2 + (\sigma_{3-4})^2} = \sigma_{TE} \text{ for event no. } \underline{\quad}$$



$$\sqrt{(\sigma_{1-2})^2 + (\sigma_{2-3})^2 + (\sigma_{3-4})^2} = \sigma_{TE} \text{ for event no. } 4.$$

For several activities connected in series, the square root of the sum of the squares of the Activity Standard Deviations,  $\sigma$ , equals the Event Standard Deviation,  $\sigma_{TE}$ , of the event terminating the series.

$$\text{Symbolically, } \sqrt{(\sigma_{1-2})^2 + (\sigma_{2-3})^2 + (\sigma_{3-4})^2 + \dots} = \text{etc.} =$$

of the event terminating the series of activities.

$$\sqrt{(\sigma_{1-2})^2 + (\sigma_{2-3})^2 + (\sigma_{3-4})^2 + \dots \text{etc}} = \sigma_{TE} \text{ of the event}$$

terminating the series of activities.



For activities connected in series, as shown below:

$$\sqrt{(\sigma_{1-2})^2 + (\sigma_{2-3})^2 + (\sigma_{3-4})^2} = \sigma_{TE} \text{ for the event terminating the series.}$$

In the example below, the Activity Standard Deviations,  $\sigma$ , of the activities are respectively:

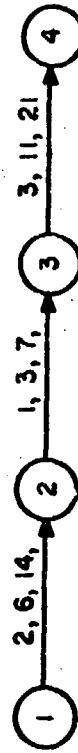
$$\sigma_{1-2} = (14 - 2)/6 = 2.0; \sigma_{2-3} = (7 - 1)/6 = 1.0; \sigma_{3-4} = (21 - 3)/6 = 3.0.$$

$$\text{Squaring these values, } (\sigma_{1-2})^2 = 4; (\sigma_{2-3})^2 = 1; (\sigma_{3-4})^2 = 9.$$

$$\text{Adding these squares, } 4 + 1 + 9 = 14.$$

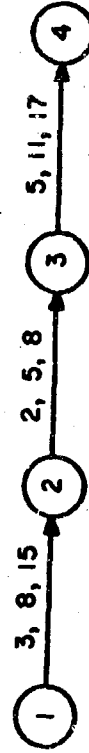
The square root of 14 = 3.7. (See Section 3, Square and Square Root Table. Panel G page 6-7 in your workbook).

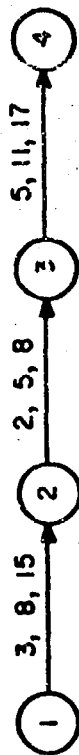
Therefore, the Event Standard Deviation,  $\sigma_{TE}$ , of event No. 4 is \_\_\_\_\_.



The Event Standard Deviation,  $\sigma_{TE}$ , of event No. 4 is 3.7.

For the three activities connected in series below  
 $\sigma_{TE}$  of event No. 4 = \_\_\_\_\_.





$\sigma_{TE}$  of event No. 4 = 3.0.

If you got this answer, turn to the next frame. If not, read on.

$$(b - a)/6 = \sigma$$

$$\sigma_{1-2} = (15 - 3)/6 = 12/6 = 2.0; (\sigma_{1-2})^2 = 4.0$$

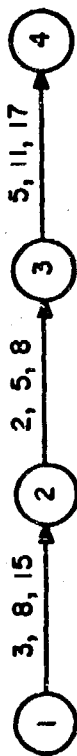
$$\sigma_{2-3} = (7 - 1)/6 = 6/6 = 1.0; (\sigma_{2-3})^2 = 1.0$$

$$\sigma_{3-4} = (17 - 5)/6 = 12/6 = 2.0; (\sigma_{3-4})^2 = 4.0$$

$$\text{Thus, } (\sigma_{1-2})^2 + (\sigma_{2-3})^2 + (\sigma_{3-4})^2 = 9.$$

Taking the square root of both sides:

$$\sqrt{(\sigma_{1-2})^2 + (\sigma_{2-3})^2 + (\sigma_{3-4})^2} = \sqrt{9} = 3 = \sigma_{TE}$$



$\sigma_{TE}$  of event No. 4 = 3.0.

If you got this answer, turn to the next frame. If not, read on.

$$(b - a)/6 = \sigma$$

$$\sigma_{1-2} = (15 - 3)/6 = 12/6 = 2.0; (\sigma_{1-2})^2 = 4.0$$

$$\sigma_{2-3} = (7 - 1)/6 = 6/6 = 1.0; (\sigma_{2-3})^2 = 1.0$$

$$\sigma_{3-4} = (17 - 5)/6 = 12/6 = 2.0; (\sigma_{3-4})^2 = 4.0$$

$$\text{Thus, } (\sigma_{1-2})^2 + (\sigma_{2-3})^2 + (\sigma_{3-4})^2 = 9.$$

Taking the square root of both sides:

$$\sqrt{(\sigma_{1-2})^2 + (\sigma_{2-3})^2 + (\sigma_{3-4})^2} = \sqrt{9} = 3 = \sigma_{TE}$$

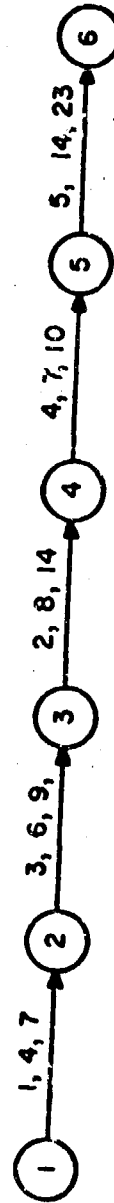
The Event Standard Deviation,  $\sigma_{TE}$ , of an event which terminates several activities connected in series equals the square root of the sum of the squares of the Activity Standard Deviations,  $\sigma$ . If this series of activities forms the critical path leading from the network beginning event to the network ending event, the Event Standard Deviation is that of the

network ending event

The Event Standard Deviation,  $\sigma_{TE}$ , of the network ending event is derived from the Activity Standard Deviations,  $\sigma$ , of the \_\_\_\_\_ path.

critical

The critical path of a certain network is shown below. The  $\sigma_{TE}$  of network ending event, No. 6, equals \_\_\_\_\_.



The  $\sigma_{TE}$  of network ending event, No. 6, equals 4.0.

If you got this answer, turn to the next page. If not, read on.

$(b - a)/6 = \sigma$			
Activity	$\sigma$	$\sigma$	$\frac{2}{\sigma}$
1-2	1	1	1
2-3	1	1	1
3-4	2	2	4
4-5	1	1	1
5-6	3	3	9
		$\frac{16}{16} = \text{Sum of } \sigma^2$	

Square root of sum of  $\sigma^2 = 4 = \sigma_{TE}$

Find your error, correct your answer, then turn to the next page.



Although the value of  $\sigma$  need only be calculated to the first decimal place for each activity, the square of  $\sigma$  ( $\sigma^2$ ) should be calculated to the second decimal place when these squares are to be added. Also  $\sigma^2 E$ , the square root of the resulting sum, need only be calculated to the first decimal place.

Section 3 of Panel G of the workbook is a table of useful square root and square values. Turn to it.

Example: For a certain critical path 1-2-3-4-5:

<u>Activity</u>	<u><math>\sigma</math></u>	<u><math>\sigma^2</math></u>
1-2	1.2	1.44
2-3	2.3	5.29
3-4	1.5	2.25
4-5	1.8	3.24
		<u>12.22</u> sum of $\sigma^2$

$\sigma_{TE}$  of event no. 5 = the square root of 12.22. From section 3, Panel G, we see that the square root of 12.0, to the first decimal place, equals 3.5 and the square root of 13 is 3.6. Since we need only the first decimal place for  $\sigma_{TE}$ , the square root of 12.22 can be taken as 3.5. Consequently  $\sigma_{TE} = 3.5$  for event no. 5.



In the above activity path  $\sigma_{TE}$  of event no. 5 equals \_\_\_\_\_.

The  $\sigma_{TE}$  of event no. 5 = 4.1.

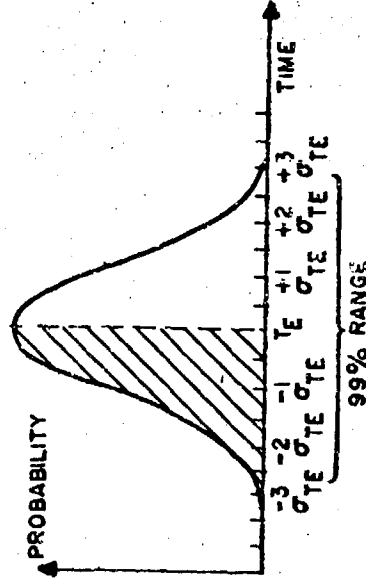
If you got this answer, turn to the next page. If not, correct your answer, then read below.

$\sigma = \frac{b-a}{6}$	Activity	$b-a$	$\sigma$	$\sigma^2$
	1-2	8.4	1.4	1.96
	2-3	7.2	1.2	1.44
	3-4	13.8	2.3	5.29
	4-5	17.4	2.9	8.41
				<u>17.10</u>

From section 3, Panel G, the square root of 17.0 is 4.1 and the square root of 18.0 is 4.2 to the first decimal place. The square root of 17.10 must equal 4.1. This is the value of  $\sigma_{TE}$  for event no. 5.

Knowing the  $T_E$  and  $\sigma_{TE}$  values of a network ending event permits us to calculate the probability of accomplishing this event, and thereby completing the network, by any given Schedule Date,  $T_S$ .

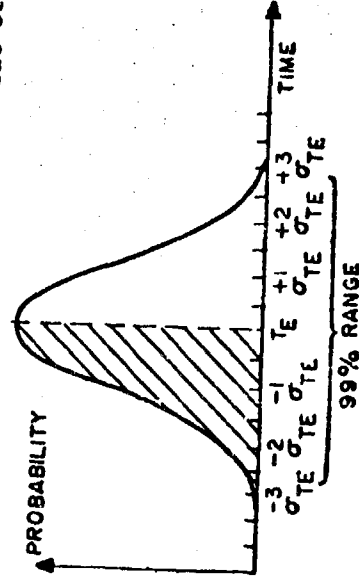
Together  $T_E$  and  $\sigma_{TE}$  define a probability curve whose shape approaches the normal distribution shown below, particularly if ten or more activities compose the critical path. The range extending from  $-3\sigma_{TE}$  on the left of the mean value,  $T_E$ , to  $+3\sigma_{TE}$  on the right of the  $T_E$  includes 99% of all the elapsed times at which it is possible for the event to occur. The vertical height of any point of the curve is a measure of the probability that the event will occur at the corresponding time of the horizontal axis. The area to the left of any vertical line is the probability that the event will occur on or before the corresponding time.



In effect the horizontal width of the probability curve, extending from  $-3\sigma_{TE}$  to  $+3\sigma_{TE}$ , measures the uncertainty of the  $T_E$  value. As  $\sigma_{TE}$  becomes smaller, the horizontal width narrows and the more certain does  $T_E$  become; as  $\sigma_{TE}$  becomes larger,  $T_E$  becomes less certain.

For example, suppose the  $T_E$  of a network ending event equals 15 weeks and  $\sigma_{TE}$  for this event equals 1.0 week. Then there is a small probability that the network either may be completed in as little time as 12 weeks ( $-3\sigma_{TE}$ ) or may require as long as 18 weeks ( $+3\sigma_{TE}$ ). However, if  $\sigma_{TE}$  equals 2.0 weeks, it is possible for the network to be completed in 9 weeks or to require as long as 21 weeks.

As  $\sigma_{TE}$  becomes smaller, the possible difference of event occurrence from the  $T_E$  value becomes less; that is, the calculated value of  $T_E$  becomes more certain.



$\sigma_{TE}$  is derived from a series of  $\sigma^2$  values which in turn are dependent on the quantity  $(b - a)/6$  for each of several activities.

To maximize the certainty of  $T_E$  for the network ending event, the quantity  $(b - a)$ , the difference between the Pessimistic Time, b, and the Optimistic Time, a, estimated for each activity, should be:

Turn to page

- |                          |       |
|--------------------------|-------|
| A. As large as possible. | 2-161 |
| B. As small as possible. | 2-162 |

Your Answer: A. For each activity,  $(b - a)$  should be as large as possible to maximize the certainty of  $T_E$ .

Sorry. Remember  $T_E$  becomes more certain as  $\sigma_{TE}$  becomes smaller.  $\sigma_{TE}$  is the square root of the sum of the  $\sigma^2$  values. This sum decreases as the Activity Standard Deviations ( $\sigma$ ) take smaller values. Since each  $\sigma$  value equals  $(b - a)/6$ , you can see that only small values of  $(b - a)$  will result in the desired small value of  $\sigma_{TE}$ .

Return to page 2-160, review it, then select the right answer.



Your Answer: B. For each activity,  $(b - a)$  should be as small as possible to maximize the certainty of  $T_E$ .

Right!  $T_E$  becomes more certain as  $\sigma_{TE}$  becomes smaller.  $\sigma_{TE}$  equals the square root of the sum of the  $\sigma^2$  values, and  $\sigma = (b - a)/6$ . Therefore,  $\sigma_{TE}$  will have the small value desired to maximize  $T_E$  certainty only if the  $(b - a)$  values are themselves small.

It may not be possible to keep  $(b - a)$  small in all cases, but at least  $(b - a)$  values that appear abnormally large when compared to others of the network should be examined as possible errors in judgment.

If  $T_E$  and  $\sigma_{TE}$  of an event (particularly a network ending event) are known, we can calculate the probability that the event will occur by its Latest Time,  $T_L$ . To do this we use the equation,

$$Z = \frac{T_L - T_E}{\sigma_{TE}}$$

In this equation  $T_L - T_E$  equals the Slack,  $S$ , of the event, as discussed previously. If a calendar Schedule Date,  $T_S$ , is assigned to the event,  $T_L$  is the elapsed time corresponding to this calendar date. If no Schedule Date is assigned,  $T_L$  is obtained by subtracting  $t_e$  values from the  $T_L$  of the network ending event. The  $T_L$  of the network ending event equals the elapsed time corresponding to its  $T_S$  calendar date. If no  $T_S$  is assigned to the network ending event, its  $T_L$  value equals its  $T_E$  value, the Accumulated Expected Time of the event.

In the formula  $Z = \frac{T_L - T_E}{\sigma_{TE}}$ ,  $Z$  is an estimate of probability that is

used with panel G. Open the workbook to Panel G.

Section 1 of Panel G is to be used with \_\_\_\_\_ values of Z.

Section 2 of Panel G is to be used with \_\_\_\_\_ values of Z.

For either section Z = \_\_\_\_\_.

By means of this equation we determine the probability of an event occurring by its \_\_\_\_\_ ( $T_L/T_E$ ) time.

When Z is negative  $T_L$  must be \_\_\_\_\_ (less/greater) than  $T_E$ .

When Z is positive  $T_L$  must be \_\_\_\_\_ (less/greater) than  $T_E$ .

positive

negative

$$Z = \frac{T_L - T_E}{\sigma T_E}$$

$T_L$

less

greater

In Panel G, values of Z are given to \_\_\_\_\_ (one/three) decimal place(s).

Z = 1.1, Probability P = \_\_\_\_\_.

Z = -1.1, Probability P = \_\_\_\_\_.

Z = 0.5, P = \_\_\_\_\_.

Z = -0.5, P = \_\_\_\_\_.

Z = 2.3, P = \_\_\_\_\_.

Z = -2.3, P = \_\_\_\_\_.

Z

1.1
-1.1
0.5
-0.5
2.3
-2.3

one

P

0.864
0.136
0.692
0.309
0.989
0.011

The value of Z need be calculated to only the first decimal place.

(See page 2-137 for a review of how to round off numbers to the first decimal place.)

Suppose that for a certain event,  $T_L = 17.0$  weeks,  $T_E = 13.5$  weeks, and  $\sigma_{TE} = 2.0$  weeks.

$$Z = \frac{T_L - T_E}{\sigma_{TE}} = \frac{17.0 - 13.5}{2.0} = \frac{3.5}{2} = 1.8$$

Z is positive. Therefore, we use Section 1 of Panel G. Here, next to  $Z = 1.8$ , we find the probability value 0.964. This means that there are 96.4 chances out of 100 that the event will be accomplished within 17.0 weeks after the network beginning event takes place.

Suppose  $T_L = 15.5$  weeks,  $T_E = 13.5$  weeks, and  $\sigma_{TE} = 2.0$  weeks. The probability that the event will be accomplished within 15.5 weeks after the network beginning event occurs is \_\_\_\_\_.

Probability equals 0.841.

$$Z = \frac{T_L - T_E}{\sigma_{TE}} = \frac{15.5 - 13.5}{2.0} = \frac{2.0}{2.0} = 1.0$$

In Section 1 of Panel G next to  $Z = 1.0$  is the probability value 0.841.



For a certain event  $T_L = 17.5$  weeks,  $T_E = 19.1$  weeks, and  $\sigma_{TE} = 1.2$  weeks.

$$Z = \frac{T_L - T_E}{\sigma_{TE}} = \frac{17.5 - 19.1}{1.2} = \frac{-1.6}{1.2} = -1.3$$

$Z$  is negative. Therefore we use section 2 of Panel G. Here next to  $Z = -1.3$  we find the probability value 0.097. There are 9.7 chances out of a hundred of this event occurring within 17.5 weeks after the network beginning event.

Suppose  $T_L = 15.9$  weeks,  $T_E = 18.2$  weeks, and  $\sigma_{TE} = 2.6$  weeks. The probability that the event will occur within 15.9 weeks after the network beginning event is \_\_\_\_\_.

The probability that the event will occur within 15.9 weeks after the network beginning event is 0.184.

$$Z = \frac{T_L - T_E}{\sigma_{TE}} = \frac{15.9 - 18.2}{2.6} = \frac{-2.3}{2.6} = -0.88 = -0.9$$

In Section 2 of Panel G next to  $Z = -0.9$  is the probability value of 0.184.

Determine the probability for the following.

Event: $T_L$ (weeks)	Event: $T_E$ (weeks)	Event: $\sigma_{TE}$ (weeks)	Probability of event occurring by its $T_L$ time
18.3	17.3	1.4	_____
23.6	19.7	2.1	_____
15.5	16.9	1.7	_____
14.7	17.0	1.3	_____

Event $T_L$ (weeks)	Event $T_E$ (weeks)	Event $\sigma_{TE}$ (weeks)	Probability of event occurring by its $T_L$ time
18.3	17.3	1.4	<u>0.758</u>
23.6	19.7	2.1	<u>0.971</u>
15.5	16.9	1.7	<u>0.212</u>
14.7	17.0	1.3	<u>0.036</u>

Work on a certain project is to start on 1 January 1962, and the Schedule Date for its final event is 3 December 1962. The Accumulated Expected Time of the final event is 47.0 weeks, and its Event Standard Deviation,  $\sigma_{TE}$ , is 2.0 weeks. The probability that the network will be completed by 3 December 1962 is \_\_\_\_\_.

Probability equals 0.692

$T_L$  corresponding to 3 December 1962 is 48 weeks.

$$Z = \frac{T_L - T_E}{\sigma_{TE}} = \frac{48 - 47}{2} = 0.50$$

From Section I of Panel C

If  $Z = 0.50$ , then probability = 0.692.

If the  $T_L$  of a network ending event equals its  $T_E$  value, the probability that the network will be completed on time is \_\_\_\_\_.

Probability = 0.500

$$Z = \frac{T_L - T_E}{\sigma_{TE}} = \frac{0}{\sigma_{TE}} = 0$$

From Section 1, Panel G, if  $Z = 0$ , probability = 0.500.

Suppose a network is to start on 1 January 1962 and is scheduled to be completed by 10 December 1962. The  $T_E$  of the network ending event is 51.0 weeks, and its  $\sigma_{TE}$  is 2.0 weeks. The probability that the network will be completed by 10 December 1962 is \_\_\_\_\_.

Probability' = 0.159

$T_L$  corresponding to 10 December 1962 is 49 weeks.

$$Z = \frac{T_L - T_E}{\sigma_{TE}} = \frac{49 - 51}{2.0} = \frac{-2}{2} = -1.0$$

From Section 2 of Panel G, when  $Z = -1.0$  probability = 0.159.

---

$$Z = \frac{T_L - T_E}{\sigma_{TE}} \quad \text{Slack, } S = T_L - T_E$$

Therefore  $Z =$  \_\_\_\_\_ divided by  $\sigma_{TE}$

Z = Slack (or S) divided by  $\sigma_{TE}$

$$Z = \frac{S}{\sigma_{TE}}$$



$$Z = \frac{T_L - T_E}{\sigma_{TE}} = \frac{S}{\sigma_{TE}} \quad \text{If, in this equation, } T_L - T_E \text{ or } S \text{ is expressed as}$$

a multiple of  $\sigma_{TE}$ , we can see the effect of Slack ( $S = T_L - T_E$ ) on the probability of accomplishing an event within the elapsed time corresponding to its  $T_L$  value. Suppose that for a certain event  $T_L = 15.0$  weeks,  $T_E = 19.5$  weeks, and  $\sigma_{TE} = 1.5$  weeks. Then  $T_L - T_E = S = -4.5$  weeks  $= -3\sigma_{TE}$ .

$$Z = \frac{S}{\sigma_{TE}} = \frac{-3\sigma_{TE}}{\sigma_{TE}} = -3$$

From Section 2 of Panel G the probability corresponding to  $Z = -3$  is 0.001. This means that if  $T_L - T_E = -3\sigma_{TE}$ , there is only one chance in 1000 that the event will be accomplished within 15.0 weeks.

Find the probability corresponding to each of the  $\sigma_{TE}$  multiples of Slack shown below.

<u>S</u>	<u>Probability</u>
$-3\sigma_{TE}$	<u>0.001</u>
$-2\sigma_{TE}$	<u>          </u>
$-1\sigma_{TE}$	<u>          </u>
$0\sigma_{TE}$	<u>          </u>
$+1\sigma_{TE}$	<u>          </u>
$+2\sigma_{TE}$	<u>          </u>
$+3\sigma_{TE}$	<u>          </u>

<u>S</u>	<u>Probability</u>
$-3\sigma_{TE}$	0.001
$-2\sigma_{TE}$	0.023
$-1\sigma_{TE}$	0.159
$0\sigma_{TE}$	0.500
$+1\sigma_{TE}$	0.841
$+2\sigma_{TE}$	0.977
$+3\sigma_{TE}$	0.999

$S$	Probability	$Z = \frac{S}{\sigma_{TE}}$
$-3\sigma_{TE}$	0.001	
$-2\sigma_{TE}$	0.023	
$-1\sigma_{TE}$	0.159	
$0\sigma_{TE}$	0.500	
$+1\sigma_{TE}$	0.841	
$+2\sigma_{TE}$	0.977	
$+3\sigma_{TE}$	0.999	

Assume that we are considering a network ending event. From this table we see that as  $S$  equals increasing and positive multiples of  $\sigma_{TE}$ , the probability of completing the network on schedule increases.

However experience shows that if Slack ( $S = T_L - T_E$ ) is negative and exceeds  $-1\sigma_{TE}$ , you can be reasonably sure that completing the network on time will be a problem. And if Slack exceeds  $-2\sigma_{TE}$ , you can be darn sure there will be problems.

In short, if a manager thinks of the Slack of the network ending event in terms of the Event Standard Deviation,  $\sigma_{TE}$ , he has a measure of the probability that the network will be completed on schedule.

# TEST NO. 5

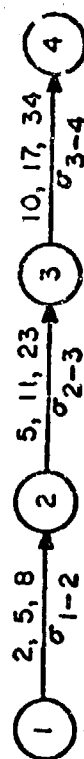
Circle the letter identifying the answer which appears to be most nearly correct.

1. The formula for the activity standard deviation is:

- a)  $\sigma = (a-b)/6$
- b)  $\sigma = (b-m)/6$
- c)  $\sigma = (b-a)/6$
- d)  $\sigma = (m-a)/6$

2. In the network shown below what is  $\sigma$  for activity 1-2?

- a) 6
- b) 1
- c) 5
- d) 8



3. The event standard deviation for event 4 in the network shown in question 2 is:

- a) 26.0
- b) 18.2
- c) 5.1
- d) 10.3

4. If Z equals 0, probability equals:

- a) 0.001
- b) 0.999
- c) 0.341
- d) 0.500

5. The formula for  $\sigma_{TE}$  in the figure in question 2 is:

- a)  $\sigma_{TE} = \sqrt{\sigma_{1-2}^2 + \sigma_{2-3}^2 + \sigma_{3-4}^2}$
- b)  $\sigma_{TE} = (\sigma_{1-2})^2 + (\sigma_{2-3})^2 + (\sigma_{3-4})^2$
- c)  $\sigma_{TE} = \sqrt{(\sigma_{1-2} + \sigma_{2-3} + \sigma_{3-4})^2}$
- d)  $\sigma_{TE} = \sqrt{(\sigma_{1-2})^2 + (\sigma_{2-3})^2 + (\sigma_{3-4})^2}$

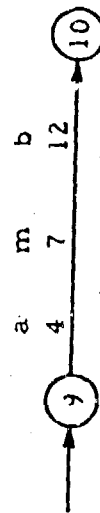
If you had difficulty selecting the right answers, review chapter 5.

## Chapter 5

### Summary

Because of the uncertainties associated with  $t_e$  and  $T_E$ , the manager must be concerned with the probabilities that events, particularly the network ending event, will occur at the times calculated. Statistical information can provide a measure of the risks involved in trying to meet a schedule.

The time span between pessimistic and optimistic time for each activity is expressed by  $(b-a)/6$ , a quantity called  $\sigma$  or Activity Standard Deviation. In the activity shown below, for example,  $\sigma = 1.3$ .



The value of  $m$ , Most Likely Time, never affects the value of  $\sigma$ . The standard deviation of an event terminating a given activity path is called Event Standard Deviation or  $\sigma_{TE}$ .

$$\sigma_{TE} = \sqrt{\sigma_1^2 + \sigma_2^2 + \sigma_3^2 + \dots + \sigma_n^2}$$

where  $\sigma_1$ ,  $\sigma_2$ ,  $\sigma_3$ , etc., are the standard deviations of the activities on the path.

As with the estimates on which they are based,  $\sigma$  and  $\sigma_{TE}$  are expressed in units and tenths of a 7-day week.

$\sigma_{TE}$  of the network ending event is derived from the standard deviations of the activities on the critical path.

To calculate the probability that a given event will occur by a given  $T_S$ , use the equation

$$Z = \frac{T_L - T_E}{\sigma_{TE}}$$

Here  $T_L$  is the elapsed time between the  $T_S$  of the given event and the  $T_S$  of the network beginning event where  $T_S - T_E$  is the slack of the event, and  $Z$  is a parameter of probability that is used with a normal probability table. Replace  $T_S$  by  $T_L$  if no  $T_S$  is specified.



PER Teach

Volume II

CHAPTER 6

Review of PERT Fundamentals

The critical path of the PERT network for a small project is shown below, together with the three estimated times (in weeks) for each of the critical path activities. The project is to be started on 1 February 1962 and is scheduled to be completed by 30 June 1962. We are to determine the probability that the project will be completed on time.



Without involving any network characteristics we can immediately determine the  $T_L$  of the network ending event. The Schedule Date  $T_S$  of the network ending event is 30 June, 1962. The  $T_S$  of the network beginning event is 1 February, 1962. Therefore, the  $T_L$  of network ending event, no. 5, is \_\_\_\_\_ weeks.

The  $T_L$  of network ending event, no. 5, is 21.4 weeks.

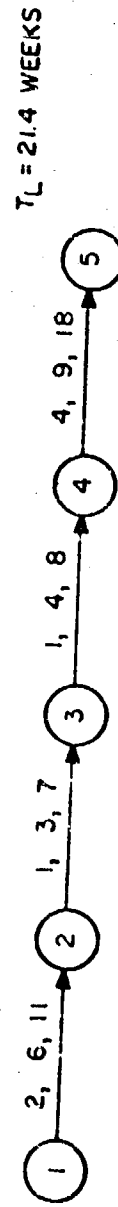
If you obtained this answer turn to the next page. If not, read on.

Latest Time,  $T_L$ , is, in effect, the time allowed to complete the program.  $T_L$  is the elapsed time between Schedule Date,  $T_S$ , of the network ending event and  $T_S$  of the network beginning event. To determine  $T_L$  for our network ending event, no. 5, compute the number of days between the two  $T_S$  dates. Divide the result by 7 to express  $T_L$  in weeks.

Month	Days	If you wish to review Latest Time, $T_L$ , and Schedule Date $T_S$ , read chapters 1 and 3 again.
Feb	28	
March	31	
April	30	
May	31	
June	30	
	<u>150</u>	
	days.	$150/7 = 21.4$ weeks

Correct your answer and then turn to the next page.

We have found that 21.4 weeks is the  $T_L$  of our network ending event. This is the elapsed time allowed to complete the project. Now we will find the elapsed time required for the work.



To find this total elapsed time, we must first determine the Expected Activity Time,  $t_e$ , for each activity.

The Expected Activity Time,  $t_e$ , for activity 1-2 is \_\_\_\_\_ weeks.

The Expected Activity Time,  $t_e$ , for activity 1-2 is 6.2 weeks.

If you obtained this answer turn to the next page. If not, correct your answer, then read the rest of this page.



$$t_e = \frac{a + 4m + b}{6} \quad \text{For activity 1-2, } t_e = \frac{2 + 24 + 11}{6} = 6.2$$

If you wish to review Expected Activity Time,  $t_e$ , read chapter 4 of Volume I again. To review activity time estimates, read chapter 3 of Volume I.

$T_L = 21.4$  WEEKS



For this critical path the Accumulated Expected Time,  $T_E$ , of event no. 5,  
the network ending event, equals \_\_\_\_\_ weeks.

The  $T_E$  of event no. 5 equals 23.4 weeks.

If you obtained this answer turn to page 2-195.

If not, correct your answer, then turn to page 2-193.

$T_L = 21.4$  WEEKS

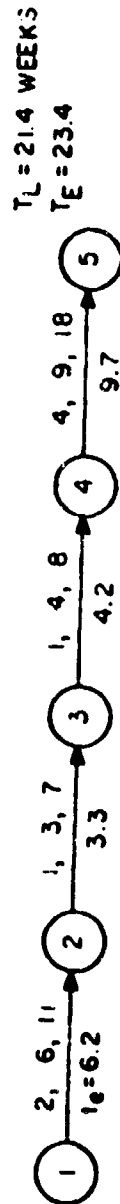


The  $T_E$  of the network ending event, no. 5, equals the sum of the  $t_e$  values of the activities composing the critical path. To find this  $T_E$ , complete the following table.

Activity	$t_e$
1-2	6.2 weeks
2-3	_____
3-4	_____
4-5	_____
	_____ = sum of $t_e$ values = $T_E$ of event no. 5.

Now turn to page 2-194.





$$t_e = \frac{a + 4m + b}{6} = (a + 4m + b)/6 \text{ for each activity.}$$

Activity	$t_e$
1-2	$\frac{6.2}{\text{weeks}} = (2 + 24 + 11)/6$
2-3	$\frac{3.3}{\text{weeks}} = (1 + 12 + 7)/6$
3-4	$\frac{4.2}{\text{weeks}} = (1 + 16 + 8)/6$
4-5	$\frac{9.7}{\text{weeks}} = (4 + 36 + 18)/6$

The sum of these  $t_e$  values is 23.4 weeks, the  $T_E$  of network ending event no. 5.

To review Accumulated Expected Time,  $T_E$ , read chapter 6 of Volume I again.  
 To review Critical Path, read chapter 7 of Volume I.

We have found that the  $T_L$  of event no. 5 is 21.4 weeks and that the  $T_E$  of this event is 23.4 weeks.

S, the slack of event no. 5 is \_\_\_\_\_ (positive/negative) and is equal to \_\_\_\_\_ weeks.

S, the Slack of event no. 5 is negative and is equal to 2.0 weeks.

$$S = -2.0 \text{ weeks.}$$

If you obtained these answers turn to the next page. If not, correct your answers, then read the rest of this page.

$$\text{Slack, } S, = T_L - T_E$$

$$\text{For our example, } S = 21.4 - 23.4 = -2.0$$

To review Slack, read Chapters 2 and 4 again.



We must now determine the squares of the Activity Standard Deviations,  $\sigma$ , in order to find the Event Standard Deviation  $\sigma_{TE}$  of event no. 5.

The square of the Activity Standard Deviation,  $\sigma$ , for activity 1-2 equals

\_\_\_\_\_.

The square of Activity Standard Deviation,  $\sigma$ , for activity 1-2 equals 2.25.

If you obtained this answer, turn to the next page. If not, correct your answer, then read the rest of this page.



Activity Standard Deviation  $\sigma = (b - a)/6$ . For activity 1-2,  $(b - a)/6 = 1.5$ .  
The square of 1.5 = 2.25.

To review Activity Standard Deviation read pages 2-133 through 2-139.

$T_L = 21.4$  WEEKS

$T_E = 23.4$   
 $S = -2$



$\sigma_{TE}$  for event no. 5 equals \_\_\_\_\_ weeks.

$\sigma_{TE}$  for event no. 5 equals 3.2 weeks.

If you obtained this answer, turn to page 2-203.

If not, correct your answer then turn to page 2-201.

$T_L = 21.4$  WEEKS

$T_E = 23.4$

$S = -2$



$\sigma_{TE}$  for event no. 5 equals  $\sqrt{(\sigma_{1-2})^2 + (\sigma_{2-3})^2 + (\sigma_{3-4})^2 + (\sigma_{4-5})^2}$ .

To find  $\sigma_{TE}$ , complete the following table:

Activity	$(b - a)/6$	$=$	$\sigma$	$\sigma^2$
1-2	$(11-2)/6$		1.5	2.25
2-3	$(7-1)/6$		1.0	
3-4	$(8-1)/6$		1.2	
4-5	$(18-4)/6$		2.3	

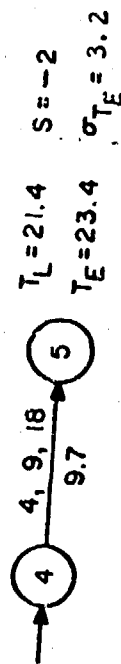
$\sigma_{TE} = \sqrt{\text{sum of } \sigma^2} = \underline{\hspace{2cm}}$



<u>Activity</u>	<u><math>\sigma^2</math></u>
1-2	2.25
2-3	1.00
3-4	1.44
4-5	5.29
	<u>9.98</u> = sum of $\sigma^2$

For event no. 5,  $\sigma_{TE} = \sqrt{\text{sum of } \sigma^2} = \sqrt{9.98} = 3.2$  weeks for event no. 5.

To review  $\sigma_{TE}$  read pages 2-140 through 2-162 again.



Having found  $\sigma_{T_E}$  of event no. 5, we can now determine the probability that this event (and therefore the entire network) will be completed within the allowed 21.4 weeks.

The probability that the network will be completed within 21.4 weeks is \_\_\_\_\_.

The probability that the network will be completed within 21.4 weeks is 0.274.  
If you got this answer turn to the next page. If not correct your answer then read below.

$$Z = \frac{T_L - T_E}{\sigma_{TE}} = \frac{21.4 - 23.4}{3.16} = -0.6.$$

From Section 2 of Panel G, if  $Z = -0.6$ , then probability = 0.274.

To review  $Z$  and probability, read pages 2-163 through 2-185.

PER Teac

Volume II

CHAPTER 7

Network Replanning and/or Simulation

2-205

The chances of completing a network on schedule can be determined fairly well from the Slack value of the critical path, and can be calculated more precisely by means of probability computations. From this information management may decide that the risk of being late is too great or may, for other reasons, believe that a given completion date is unrealistic. In any of these cases, a PERT network may often be modified or "replanned" to improve the chances of completing the network within the time allowed.

Replanning is an area that illustrates the flexibility of the PERT system and which calls for managerial decision. There are only three types of action a manager may take to replan a network, but the type to be used depends on circumstances.

A network may be replanned by one or more of the following three methods:

1. Changing a chain of series-connected activities into a series-parallel arrangement.
2. Changing resources applied to activities.
3. Changing the scope of various activities, even eliminating activities as a last resort.

The three replanning procedures outlined above have one purpose in common. They are intended to:

Your Answer

Turn to page

A. Increase the time allowed to complete the network.

2-208

B. Decrease the time required to complete the network.

2-209

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Your Answer: A. Replanning procedures are intended to increase the time allowed to complete a network.

Wrong. If 50 weeks are allowed for a program that requires 57, changing the network affects the time required, not that allowed. The time allowed to accomplish work is physically independent of the time required.

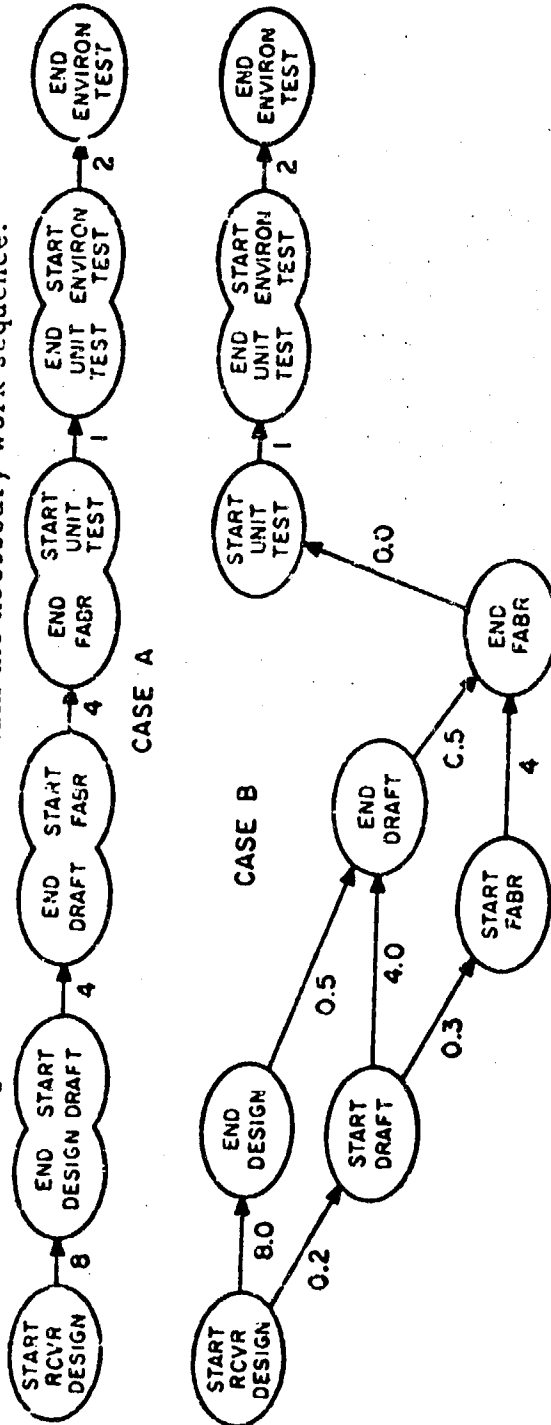
Read page 2-207 again and then select the correct answer.

Your Answer: B. Replanning procedures are intended to decrease the time required to complete a network.

Correct! Modification of PERT activities usually affects the time required to complete them. To improve the chances of meeting a given completion date, always try to reduce such time.



One method of reducing the time allowed is to transform a set of series connected activities into a series-parallel arrangement. The following elementary sketch shows how this is done. In case B, note that each activity of work keeps its beginning and ending events, and is joined to others by short constraining activities which maintain the necessary work sequence.



All  $t_e$  values written below the activities are in units and tenths of a week. In Case A, the activity path extending from "start receiver design" to "end environment test" is expected to require a total of \_\_\_\_\_ weeks. In Case B where the same activities now include a series-parallel arrangement, the maximum time between these same two events is expected to be \_\_\_\_\_ weeks.

Case A, 19 weeks

Case B, 12 weeks

\_\_\_\_\_

One replanning method involves rearranging a chain of activities connected in series to include a \_\_\_\_\_ configuration.

series-parallel

or

parallel-series

Another replanning method that reduces the time required by a critical path is to increase the resources applied to various activities of this path. Such resources include manpower and equipment as well as space or capital that permit the use of more manpower and equipment. Overtime work is also a \_\_\_\_\_ that may be applied to many activities.

resource

A convenient method of obtaining increased resources for activities of the critical path is to take some resources away from other activity paths that can afford to lose them. This may increase the time durations required for these paths but as long as such durations do not closely approximate that of the critical path, the procedure is useful.

Those activity paths which can afford to lose some resources have slack values  $(T_L - T_E)$  much \_\_\_\_\_ (greater/less) than that of the critical path.

greater

Remember, the critical path always has the smallest possible slack value.

Not all activities can be shortened in time by applying increased resources to them. An engineer may design new equipment based on new principles. The time required for this activity is not easily shortened. Activities involving chemical action usually cannot be speeded up appreciably.

We \_\_\_\_\_ (can/cannot) reduce the time required to make the drawings for a new system by using more draftsmen.

We \_\_\_\_\_ (can/cannot) easily reduce the time required for concrete to harden.

can

cannot

\_\_\_\_\_

One method of replanning consists of increasing the \_\_\_\_\_ applied to  
various \_\_\_\_\_ of the \_\_\_\_\_ path.

One method of replanning consists of increasing the resources applied to various activities of the critical path.

A third method of replanning to reduce the time required by the critical path is to decrease the scope of various critical path activities, if this is feasible. By reducing the work involved in an activity, we decrease the scope of the activity.

For example, an activity may be an environmental test involving operation of a unit under these different conditions; extreme cold, extreme heat, and severe vibration. By eliminating the vibration test we reduce the \_\_\_\_\_ required to accomplish the activity.



time

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Sometimes the standards or quality assurance applying to a given activity are too high. Reducing such standards to lesser but still acceptable values is another way of reducing the work, time and \_\_\_\_\_ of the activity.

### scope

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If you plan to reduce the critical path time by reducing the scope of some of its activities, you should also consider the possibility of eliminating such activities. Maximum reduction of activity scope may be elimination of the activity.

For example suppose environmental test is an appreciable activity of the critical path. If PERT calculations show there is little chance of meeting firm delivery dates, the possibility of eliminating environmental test for the first models of the equipment should be explored.

Increasing the resources applied to an activity of the critical path will reduce the three time estimates applying to this activity. These estimates you remember, are Optimistic Time, a, Most Likely Time, m, and Pessimistic Time, b. They determine the Expected Activity Time,  $t_e$ .

Reducing the scope of the activity also reduces the values of these three time estimates and will therefore \_\_\_\_\_ (decrease/increase) the Expected Activity Time,  $t_e$ .

decrease

---

Only after increasing the resources applied to an activity or decreasing the scope of the activity are we justified in changing the three time estimates of the activity.

Changing these estimates without such justification is not considered cricket. Experience has shown that these estimates, as first provided by responsible experts who will be involved in accomplishing the activities, are as reliable as any that may be obtained later. Pressure should never be applied to make responsible personnel change their estimates in order to reduce activity times. This only reduces the effectiveness and reliability of the PERT system.

Any of the three replanning methods should be used primarily to reduce the time required by the critical path, the activity path of maximum time through the network.

However there are always several activity paths leading from the network beginning event to the network ending event. The activity path that requires maximum time after the original critical path has been replanned becomes the new critical path and determines the time required to complete the network.

Activity Path	<u>Weeks Required</u>	
	<u>Before Replanning</u>	<u>After Replanning</u>
A	39	33
B	32	32
C	37	37
D	35	35

Each Activity Path, A, B, C and D, extends from the network beginning event to the network ending event. Before replanning, Activity Path A is the critical path. After replanning, the critical path is Activity Path \_\_\_\_\_.

After replanning the critical path is Activity Path C. Activity Path C now requires maximum time, 37 weeks.

<u>Activity Path</u>	<u>Weeks Required</u>	
	<u>Before Replanning</u>	<u>After Replanning</u>
A	43	38
B	40	40
C	36	36
D	37	37

Each Activity Path, A, B, C and D, extends from the network beginning event to the network ending event. To shorten the time required to complete the network, the original critical path is replanned.

Such replanning shortens the time required to complete the network by

Your Answer  
 A. 5 weeks  
 B. 3 weeks

Turn to Page  
 2-224  
 2-225

2-223

Your Answer: A. Replanning the original critical path shortens the time needed to complete the network by 5 weeks.

Sounds logical, but this is the wrong answer.

The original critical path is the one requiring the greatest time before replanning. This is Activity Path A which requires 43 weeks. Replanning this path reduces the time it requires to 38 weeks. The difference between these two values is 5 weeks. However after replanning, Activity Path A is not the path of maximum time through the network and therefore is not the critical path.

The time needed to accomplish the entire network always equals that required by the critical path. Therefore to find the time saved by replanning we subtract the critical path time after replanning from the critical path time before replanning.

Return to page 2-223 and select the right answer.

Your Answer: B. Replanning the original critical path shortens the time needed to complete the network by 3 weeks:

Right! The original critical path, Activity Path A is shortened from 43 weeks to 38 weeks by replanning. But after replanning, this path is no longer the critical path because Activity Path B requires a greater time, 40 weeks.

The time needed to accomplish a complete network always equals the time required by the critical path. Therefore to find the time saved by replanning we subtract the critical path time after replanning from the critical path time before replanning.



Replanning may reduce the time required by the original critical path to such an extent that some other activity path requires more time and therefore becomes the new critical path. Consequently when replanning, you must always keep track of those \_\_\_\_\_ which may become \_\_\_\_\_ after replanning.

activity paths

critical paths

2-227

The activity paths which may become critical paths after replanning of the original critical path can be identified by their slack values ( $T_L - TE$ ). The critical path always has less slack than any other activity path. Those activity paths whose slack values approximate that of the original critical path are the ones likely to become critical paths after replanning.

<u>Activity Path</u>	<u>Slack</u>
A	+1
B	-3
C	0
D	-5
E	-2

In the above table Activity Path      is now the critical path. If through replanning, the slack of this path becomes equal to -1, then the new critical path will be Activity Path     .

Activity Path D is now the critical path.

After replanning, Activity Path B will be the new critical path if the slack of Activity Path D then equals -2, -1 or any other value more positive than -3.

\_\_\_\_\_

A PERT network may be replanned in one or more of the following three ways:

1. By modifying a chain of activities connected only in series so that it contains a \_\_\_\_\_ - \_\_\_\_\_ configuration of activities.
2. By increasing the \_\_\_\_\_ applied to activities of the critical path.
3. By changing the \_\_\_\_\_ of one or more activities of the critical path.

1. series-parallel

or

parallel-series

2. resources

3. scope

When undertaking replanning procedures always keep an eye on those activity paths whose \_\_\_\_\_ values approximate that of the \_\_\_\_\_ path. After replanning, one of these paths may become the new \_\_\_\_\_.

When undertaking replanning procedures always keep an eye on those activity paths whose slack values approximate that of the critical path. After replanning, one of these paths may become the new critical path.

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Simulation procedures are used to find the effects of replanning changes on the network characteristics. Simulation consists of determining the characteristics of a network which contains the proposed replanning changes. Usually the basic data (event identification and time estimates for each activity of the simulated network) are passed through the PERT computer. By means of such data processing we find the network critical path and slack values as well as the Accumulated Expected Time of the network ending event.

None of the results obtained with the \_\_\_\_\_ network apply to the real network until the proposed \_\_\_\_\_ changes have been adopted.

None of the results obtained with the simulated network apply to the real network until the proposed replanning changes have been adopted.

\_\_\_\_\_

In using the computer for simulation, the same input forms are used as are used for filing actual reports. Caution must be exercised to prevent confusion between actual data and data used for \_\_\_\_\_.

simulation

Computer printout of simulated data must be clearly identified. To prevent any accidental mixup simulated data should not be kept together with actual network data. Simulated data should be kept in a \_\_\_\_\_ file.



separate  
(or equivalent word)

Various possibilities and alternate procedures are provided by simulation.  
You as a manager must \_\_\_\_\_ which of these are to be adopted.

decide (or an equivalent word)

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Any proposed changes in the network can be simulated. Simulation can provide new outlooks on the project, and may provide important network improvements. The flexibility and usefulness of PERT are enhanced by means of \_\_\_\_\_ procedures.

simulation

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Presentation of alternatives by simulation is not a commitment. Simulation of alternatives must be carefully analyzed for the possibility of newly developed risks, or changes in the scope of the project.

Simulation is a vital aid in formulating a decision, but it is not a \_\_\_\_\_.

commitment

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When unexpected troubles occur, the computer may be used to test alternate activities, activity times, and/or events to alleviate the problem, or prove that there are no alternates. This is also called \_\_\_\_\_.

simulation

2-238

TEST NO. 6

1. The purpose of replanning is:
  - a) to increase the time allowed to complete the network.
  - b) to decrease the time required to complete the network.
  - c) to change resources.
  - d) to change activity paths.
2. To increase the probability of completing the network on time a manager may:
  - a) change resources of an activity path.
  - b) change a chain of critical path activities.
  - c) change the scope of activities.
  - d) do all of the above.
  - e) do none of the above.
3. Which statement is most nearly correct?
  - a) Only critical path proposed changes in a network can be simulated.
  - b) Any proposed change in a network can be simulated.
  - c) Only non-critical path proposed changes can be simulated.
  - d) None of the above.